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### ► To cite this version:

Nafe Moradkhani, Frederick Benaben, Benoit Montreuil, Matthieu Lauras, Thibaut Cerabona, et al.. A force-inspired paradigm for performance-based decision support-Physics of Decision application in nonlinear dynamical systems. *Journal of Industrial Information Integration*, 2024, 41, pp.100656. 10.1016/j.jii.2024.100656 . hal-04632304

**HAL Id: hal-04632304**

<https://imt-mines-albi.hal.science/hal-04632304v1>

Submitted on 10 Jul 2024

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# A force-inspired paradigm for performance-based decision support— Physics of Decision application in nonlinear dynamical systems

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## ARTICLE INFO

### Keywords:

Decision Support System (DSS)  
Performance Management  
State-space method  
Nonlinear systems  
System Dynamics  
MIMO systems  
Information Integration  
Linear Time Invariant (LTI) systems

## ABSTRACT

This paper provides a perspective on performance-based decision support. The chosen approach is based on the principles of “Physics of Decision,” which considers the performance of a system as a physical trajectory within the boundaries of its performance indicators that might be deviated through variation of system parameters. According to the overall premise of employing the state-space method to simulate physical systems, this work presents a decision aggregation method in dynamic systems. The core contribution is to propose a multi-criteria performance framework to manage multi-input-multi-output (MIMO) system performance with a combination of affordable decisions. A nonlinear inventory-workforce management model has been used to demonstrate the proposed approach.

## 1. Introduction

Providing a reliable decision support system (DSS) is a long-standing challenge in decision-making. In addition to several fundamental properties that DSS must possess (discussed in section 2), the following defects make implementing DSSs challenging and consequently present an imprecise imitation or simulation of real-world systems: (i) neglecting to consider some components with underlying effects on the system performance, (ii) improper construction of relationships between system parameters, and (iii) **inaccurate system performance predictions due to nonlinear interactions among its parameters** [1, 2, 3].

The third issue is primarily the focus of the present work, which aims to pinpoint how individual decisions in nonlinear circumstances can be combined and consequently estimate their influences on the system’s performance, despite complexities arising from its nonlinearity. To put it simply, if the studied system is nonlinear, the singular impact of its parameter variations on the system’s performance cannot be aggregated. In other words, the effects of varying single parameters and then aggregating their impacts on the system performance would not yield the same result as when the parameters vary concurrently.

In physical systems, the “explicit” method calculates the system status at a future time from the currently known system status, whereas the “implicit” methods attempt to find a solution to the nonlinear system of equations iteratively

by considering the current state of the system as well as its subsequent (or previous) time state [4]. The implicit method, for example, deals with the nonlinear relationship between applied forces on the system and its physical displacement. Figure 1 illustrates the relationships between the intensity of applied forces to an object connected to a spring (with constant and variable stiffness coefficient,  $k$ ) and the object displacement ( $F = kx$ ).

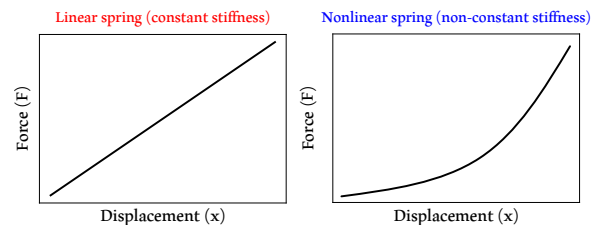


Figure 1: Mass-Spring system with linear & non-linear springs

The Physics-of-Decision (POD) is a physics-inspired framework to manage multi-input multi-output (MIMO) systems [5]. According to the original POD framework, opportunities and risks may be viewed as physical forces applied to the system trajectory that could push the system toward or away from its objectives in its performance space. The practical applications of the POD framework are diverse and impactful. For instance, in financial systems, POD can help optimize investment strategies by modeling market forces as dynamic entities. In supply chain management, it can assist in decision-making by evaluating risks and opportunities as forces affecting inventory levels and distribution efficiency. In robotics, POD principles enhance control systems, treating environmental interactions as forces that guide the robot’s actions. Leveraging the POD framework and a well-known physical system modeling, state-space representation

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[6, 7], the proposed work attempts to provide an **implicit** method to address decision-related consequences on the system performance in nonlinear dynamical systems. This paper addresses solving the nonlinearity problem through linearization methods such as the Taylor Series and Jacobian Matrix, making nonlinear dynamic systems more predictable and controllable. **The overall objective is to provide a solution to a nonlinear system to predict its performance through individual decisions by eliminating imposed consequences arising from the system's nonlinearity and assisting the system manager in evaluating several distinctive decisions before taking the best ones.**

The paper is structured as follows. Section 2 focuses on the Physics-of-Decision fundamentals and current research efforts related to the topic. Section 3 first introduces the state-space modeling, then tunes the POD framework to that. Section 4 provides two nonlinear dynamic models to present the work's significance. Finally, section 5 gives conclusive remarks and proposes areas for further research.

## 2. Background and related works

As asserted in [8], truth exists, and so does uncertainty. Uncertainty acknowledges the existence of an underlying truth that is a fact in the future. Probability, which is the language of uncertainty, therefore aims at truth. Probability presupposes truth; it is a measure or characterization of truth. Probability is not necessarily the quantification of the uncertainty of truth, because not all uncertainty is quantifiable. Uncertainties are considered potentialities in the decision-making context, whereas potentialities with undesirable outcomes are referred to as "risks" and the ones with conducive consequences as "opportunities" [5]. POD introduces an innovative DSS based on physics laws of motion to address those quantified or quantifiable potentialities that arise from incomplete information, lack of knowledge, and experimental data and are reducible in the case of risk and achievable in the case of opportunity.

### 2.1. Physics-based decision support system

Physics of Decision is concerned with multi-criteria management of systems subject to almost ubiquitous instability and uncertainty which have to be coped with, and accounted for when managing the systems and their performance [5, 9]. Decision support technologies and performance management allow the examination of such systems in the following three aspects, as proposed in [10]. **(i) Intelligence:** to comprehend the considered system, its Key Performance Indicators (KPIs), its context, its current and target performance, **(ii) Design:** to comprehend the potential changes that this system and its context may undergo, and **(iii) Choice:** to find a way to choose among changes and their effects on performance to make appropriate decisions.

Through the following methodology, the POD, an innovative approach to decision support, might completely encompass the Intelligence, Design, and Choice aspects of decision support systems in the context of instability and uncertainty [9].

- (i) **Intelligence** through system establishment and characterization: First, the intention of the considered system (objectives considered as KPIs), its specific study limits, the system's associated parameters to the recognized potential, as well as the context parameters themselves, and its internal and external (system's context) potentials are collected and described,
- (ii) **Design** through system vulnerability and sustainability identification: Second, a detailed assessment of the investigated system through sensitivity analysis (*e.g.*, simulation campaign runs, Artificial Neural Networks (ANN), and clustering) infers the correlation matrix between the system inputs and the intended KPIs,
- (iii) **Choice** through strategy exploration: Finally, regarding the system preferences and limits, using the optimization algorithms, notably the heuristic approach, the most desirable decisions are offered to lead the system toward its objectives in the performance space.

The proposed methodology is included in the POD framework, which has been studied in several contexts. Crisis management context, such as the COVID-19 pandemic crisis [9], the COVID-19 pandemic impact on air pollution [11]. Operational management contexts, such as road traffic management [12], supply chain management [13, 14], and polling place management [15, 16]. Efficiency management context, such as project management [17]. The following section describes the preliminaries and basic concepts of the POD framework.

### 2.2. Foreword, essential definitions, and notations

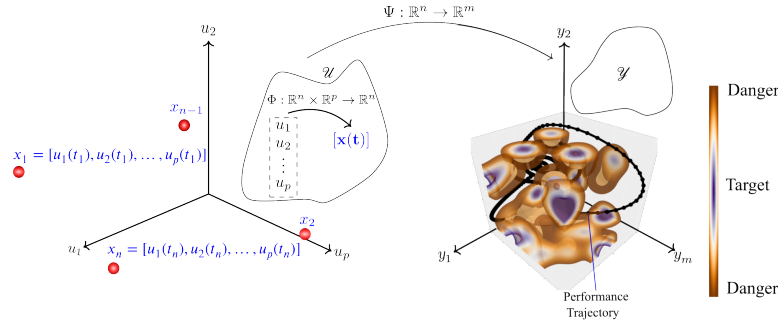
System characterization and identification are essential challenges in systems theory. The mathematical representation of a system lies at the heart of the characterization challenge. A simulation method that aims to represent the system functionality through a performance-based approach can be expressed as a function  $\mathbf{F}$  from an input space  $\mathcal{U}$ , representing the description space of the system, into an output space  $\mathcal{Y}$ , representing the performance space of the system [5]. The function  $\mathbf{F}$  is defined implicitly by the specified input-output pairs. The method of representing time-dependent systems by differential equations is well-established in systems theory and applies to a fairly large class of systems [18]. For example, the differential equation:

$$\begin{aligned} \frac{dx(t)}{dt} &\triangleq \dot{x}(t) = \Phi[x(t), u(t)], & t \in \mathbb{R}^+ \\ y(t) &= \Psi[x(t)]. \end{aligned} \quad (1)$$

where:

$$\begin{aligned} u(t) &\triangleq [u_1(t), u_2(t), \dots, u_p(t)]^T \\ x(t) &\triangleq [x_1(t), x_2(t), \dots, x_n(t)]^T \\ y(t) &\triangleq [y_1(t), y_2(t), \dots, y_m(t)]^T \end{aligned}$$

represents a  $p$ -input,  $m$ -output system of order  $n$ . While  $u_i(t)$  represents the  $i$ -th input vector at time  $t$ ,  $x_i(t)$  represents the



**Figure 2:** Mathematical representation of the Physics of Decision approach

$i$ -th state variable of inputs' vectors,  $u_1(t), u_2(t), \dots, u_p(t)$ , at time  $t$ , and  $y_i(t)$  represents the  $i$ -th output vector at time  $t$ . Functions  $\Phi$  and  $\Psi$  are dynamic<sup>1</sup> “linear” or “nonlinear” maps defined as  $\Phi : \mathbb{R}^n \times \mathbb{R}^p \rightarrow \mathbb{R}^n$  and  $\Psi : \mathbb{R}^n \rightarrow \mathbb{R}^m$ . The function  $\Phi$  presents the states of the system inputs in space  $\mathcal{U}$ ; Since the number of inputs is considerably fewer than their possible states ( $p \ll n$ ), the  $\Phi$  would be a  $n \times p \rightarrow n$  dimension function. The  $\Psi$  function is used to depict the relationship between inputs (vector  $u(t)$ ) and outputs (vector  $y(t)$ ). The vector  $x(t)$  denotes the state of the system at time  $t$  and is determined by the state at time  $t_0 < t$  and the input  $u(t)$  defined over the interval  $[t_0, t]$ . The output  $y(t)$  is determined by the state of the system at time  $t$  [19, 18]. Equation 1 is referred to the input-state-output representation of the system. This paper is concerned with dynamic systems, which can be represented by differential equations corresponding to the differential equation given in Equation 1. These take the form:

$$\begin{aligned} x(k+1) &= \Phi[x(k), u(k)], \\ y(k) &= \Psi[x(k)]. \end{aligned} \quad (2)$$

where  $u(\cdot)$ ,  $x(\cdot)$ , and  $y(\cdot)$  are discrete-time sequences. Fig. 2 depicts the POD framework as a time-dependent technique with the input-states-output vision.

According to equation 1, the variations of vector  $u(t)$  change the  $\dot{x}(t)$  via function  $\Phi$ . These changes go through function  $\Psi$  and consequently deviate the system trajectory shaped with vector  $y(t)$  in the performance space  $\mathcal{Y}$ . This transition from the variation of vector  $u(t)$  to the deviation of the system trajectory is assumed as “physical forces” in the POD framework.

### 3. Force-based paradigm in performance management

The space  $\mathcal{Y}$  in Fig. 2 presents the predetermined time-independent Target (objective) and Danger (or could be less targeted) zones concerning the intended level of quantified KPIs ( $y_1, y_2, \dots, y_m$ ) on the axes. The degree of freedom of

the system to assign values to system inputs ( $u_1, u_2, \dots, u_p$ ) in the space  $\mathcal{U}$  along with the system's sensitivity to the critical level of the system's inputs is the main factor in these determinations (system's susceptibilities) in the performance space  $\mathcal{Y}$ . Although these levels may change over time, they provide a forward-thinking view of the systems' objectives. The most essential takeaway from the presented framework in Fig. 2 is to see the studied system as an object in its multi-dimensional performance space  $\mathcal{Y}$ , which might be pushed and pulled by produced forces ( $f_1, f_2, \dots, f_p$ ) resulting from changes in the system's parameters or in its environment parameters ( $u_1, u_2, \dots, u_p$ ) at any time  $t$ . Essentially, these variations' consequences are observed through the deviation of the system "trajectory" within the space  $\mathcal{Y}$ , the multidimensional performance space of systems' KPIs.

#### 3.1. Physics of Decision in practice

The presented POD framework, emphasizes the mobility of the investigated system within its performance space  $\mathcal{Y}$ , where movements are studied in this multidimensional space defined by the system's KPIs. These KPIs evolve as a result of changes in the system's internal and contextual input parameters, vector  $u(t)$  in equation 1 [5].

The purpose of the POD theory, which derives from classical physics, is to facilitate the handling of the intricacies of the observed system to assist decision-makers in steering the system (especially in nonlinear circumstances) toward its objectives. The following illustrates the relation between a KPI (and its associated concepts) and physics notations such as displacement, velocity, and acceleration.

- (i) Displacement  $\Leftrightarrow$  Variation: The *displacement* ( $\delta x$ ) of the system on the associated axis to a KPI in the performance space  $\mathcal{Y}$  (e.g.,  $y_i$ ) is equivalent to the *variations* of that KPI ( $\delta k_i$ ),
- (ii) Velocity  $\Leftrightarrow$  Growth: The derivative of a KPI ( $\partial k_i / \partial t$ ) shows its positive/negative *growth*, which is equalized to the system's *velocity* ( $\partial x / \partial t$ ) on the KPI's connected axis,
- (iii) Acceleration  $\Leftrightarrow$  Fluctuation: The velocity derivation indicates the system's *acceleration* ( $\partial v / \partial t = \partial^2 x / \partial t^2$ ) on the connected axis.

<sup>1</sup>Although the  $\Phi$  and  $\Psi$  functions could be time-invariant (static) [9], this study concentrates mainly on dynamic (time-variant) functions

The “Displacement” of a KPI shows where the system is falling behind or veering off course to take corrective action quickly, solving the issue before it escalates into a full-blown problem. Displacement mainly gives an intuition or insight into “Operational” decisions that seek to get closer and closer to real-time measurement to evaluate the system performance on a short-term basis.

The “Velocity” of a KPI monitors the progress or trend of the system toward a stated destination (system objectives in the performance space  $\mathcal{Y}$ ) to take “Strategic” decisions over time. This measurement effectively requires observation over a more extended period to provide an accurate picture of system development.

The “Acceleration” magnitude may reflect how strongly or weakly a KPI fluctuates on its connected axis and might help on the “Operational” decisions. At the same time, its positive or negative direction may aid in the “Strategic” decision and is always the same as the net force acting on the system produced by inputs’ variations (vector  $u(t)$ ).

The displacement, velocity, and acceleration are equally important; they just provide different information for different purposes. For systems to yield all their promise, strategic and operational decisions must be aligned so that decision-makers at all levels may perceive the relationship between system activity and system performance.

The POD approach focuses on a force-based vision to assess the system’s performance. According to this vision, a system is perceived as an object, and its performance as a trajectory within the performance space  $\mathcal{Y}$  may veer off course due to internal or contextual perturbations (interpreted as risks and opportunities) viewed as physical forces acting on the object.

### 3.2. Physics of Decision installation on state-space representation

State-space representation, a well-established mathematical model in physics, provides a practical and condensed method to describe and analyze systems with multiple inputs and outputs. The most general state-space representation of a linear system with  $p$ -inputs,  $m$ -outputs, and  $n$ -state variables takes the following form:

$$\begin{aligned} \dot{x}(t) &= Ax(t) + Bu(t), \\ y(t) &= Cx(t) + Du(t) \end{aligned} \quad (3)$$

where  $A$  is the “state matrix”,  $B$  is the “input matrix”,  $C$  is the “output matrix”, and  $D$  is the “feed-through (or feed-forward) matrix”. The first equation is the *state* equation and the latter is the *output* equation. While all matrices ( $A$ ,  $B$ ,  $C$ , and  $D$ ) are allowed to be time-variant (*i.e.*, their elements can depend on time, *e.g.*,  $A(t)$ ); in the common Linear Time Invariant (LTI) systems, matrices will be time invariant (*i.e.*, the functions  $\Phi$  and  $\Psi$  in equation 1 are static). This study focuses on systems whose outputs do not influence inputs; however, this could occur in several systems (*e.g.*, thermostat heater, automatic clothes iron, etc.). In such systems, the state and output equations are influenced by a “feedback”

matrix  $K$  from the system output<sup>2</sup>. Fig. 3 shows the block diagram for the linear state-space representation, with feedback  $K$  (indicated by a dashed line) from the system’s output  $y$  to its input  $u$  [6, 7]. Using transfer functions (Laplace

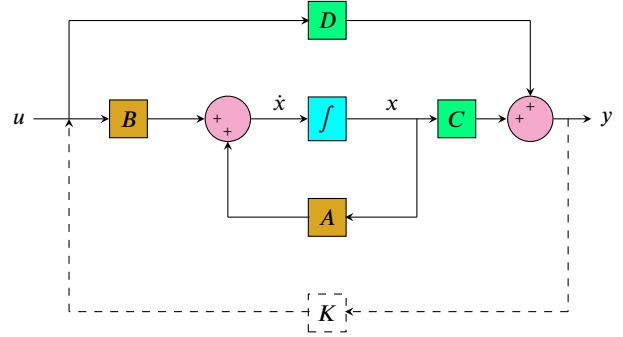


Figure 3: General state-space representation model

transfer form) or differential equations to model complex systems becomes laborious; this is even more true if the system has multiple inputs and outputs [7]. The state space method largely alleviates this complexity. Fig. 4 shows a Mass-Spring-Damper system with its mass,  $m$ , spring constant,  $k$ , and damping coefficient,  $b$ , where the external force acting on the mass,  $f(t)$ , is considered as input while its position,  $p(t)$ , is considered as output. According to the free-

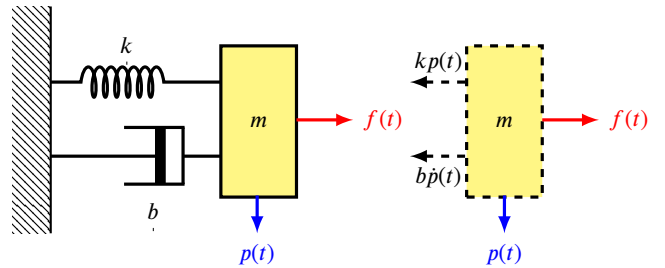


Figure 4: Mass-Spring-Damper System

body diagram shown in dashed lines and Newton’s 2nd law, equation 4 presents the differential equation of this system.

$$f(t) - b \frac{dp}{dt} - kp(t) = m \frac{d^2p}{dt^2} \quad (4)$$

This system is a straightforward illustration of a “linear” state-space representation. When a system has a mass, its position and velocity are commonly chosen as state variables. Also, position, velocity, and force (input) are sufficient to determine this system’s future position (output). For these reasons, position and velocity are chosen as state variables ( $x(t)$  in equation 3). The input vector ( $u(t)$ ), state vector ( $x(t)$ ), and output vector ( $y(t)$ ) are shown in equation 5.

$$u(t) = f(t), \quad x(t) = \begin{bmatrix} x_1(t) \\ x_2(t) \end{bmatrix} = \begin{bmatrix} p(t) \\ \dot{p}(t) \end{bmatrix}, \quad y(t) = p(t). \quad (5)$$

<sup>2</sup>In such systems, the equation 3 becomes:  $\dot{x}(t) = Ax(t) + BKy(t)$  and  $y(t) = Cx(t) + DKy(t)$  - note that  $K$  is the feedback matrix.

Considering the new notation in equation 5, equation 4 becomes equation 6.

$$u(t) - bx_2(t) - kx_1(t) = m\dot{x}_2(t) \quad (6)$$

Through rearranging equations to express  $\dot{x}(t)$  and  $y(t)$  in terms of  $x(t)$  and  $u(t)$ , equation 7 is obtained.

$$\begin{aligned} \dot{x}_1(t) &= x_2(t), \\ \dot{x}_2(t) &= -\frac{k}{m}x_1(t) - \frac{b}{m}x_2(t) + \frac{1}{m}u(t), \\ y(t) &= x_1(t). \end{aligned} \quad (7)$$

Finally, by organizing equation 7 into matrix format, equation 8 shows (the *state* and *output* equations) the state-space model of the Mass-Spring-Damper system presented in Fig. 4.

$$\begin{aligned} \begin{bmatrix} \dot{x}_1(t) \\ \dot{x}_2(t) \end{bmatrix} &= \underbrace{\begin{bmatrix} 0 & 1 \\ -k/m & -b/m \end{bmatrix}}_A \begin{bmatrix} x_1(t) \\ x_2(t) \end{bmatrix} + \underbrace{\begin{bmatrix} 0 \\ 1/m \end{bmatrix}}_B [u(t)], \\ y(t) &= \underbrace{\begin{bmatrix} 1 & 0 \end{bmatrix}}_C \begin{bmatrix} x_1(t) \\ x_2(t) \end{bmatrix}. \end{aligned} \quad (8)$$

The POD framework studies the system's trajectory in the performance space  $\mathcal{Y}$  through produced forces stemming from inputs' variations ( $u_1, u_2, \dots, u_p$ ) at any time  $t$ , as opposed to state-space representation in "Time Invariant" systems, which only has  $u(t)$  as a time-dependent force. Given this distinction, the state-space representation would be continuous time-variant, as equation 9 shows the matrices and vectors used in the POD framework could be time-variant.

$$\begin{aligned} \dot{x}(t) &= A(t)x(t) + B(t)u(t), \\ y(t) &= C(t)x(t) + D(t)u(t) \end{aligned} \quad (9)$$

Considering the whole Mass-Spring-Damper system as an object in its performance space  $\mathcal{Y}$  built on the (i) system position and (ii) system velocity as its KPIs (*i.e.*, output matrix  $C = [1 \ 1]$ ), Fig. 5 (II) presents the tuning of this system to the POD framework along with the results of several discussed investigations in the following. Considering the external force,  $f(t)$ , the mass,  $m$ , the spring constant,  $k$ , and damping coefficient,  $b$ , the input vector of this system includes  $u(t) = [f(t), m(t), k(t), b(t)] = [u_1(t), u_2(t), u_3(t), u_4(t)]$ .

To evaluate the system performance, the three following modes are considered to assess the system performance: (i) **Inertia** mode: The performance could change due to its normal behavior and the associated consumption. In this mode, the system is not facing any force, (ii) **Passive** mode: In addition to the performance changes of Inertia, the performance might change because there is (are) force(s) in the system or its environment, and (iii) **Active** mode: In addition to the performance changes of Inertia, the performance could change because the system is facing some

forces (passive changes) and also because of some taken decisions to manage the consequences of those forces [9].

*It is important to note that the parameter initialization, settings, and applied scenarios in the remaining of this section were chosen to reflect the POD framework, which could be irrational and impossible to implement in real-world circumstances.*

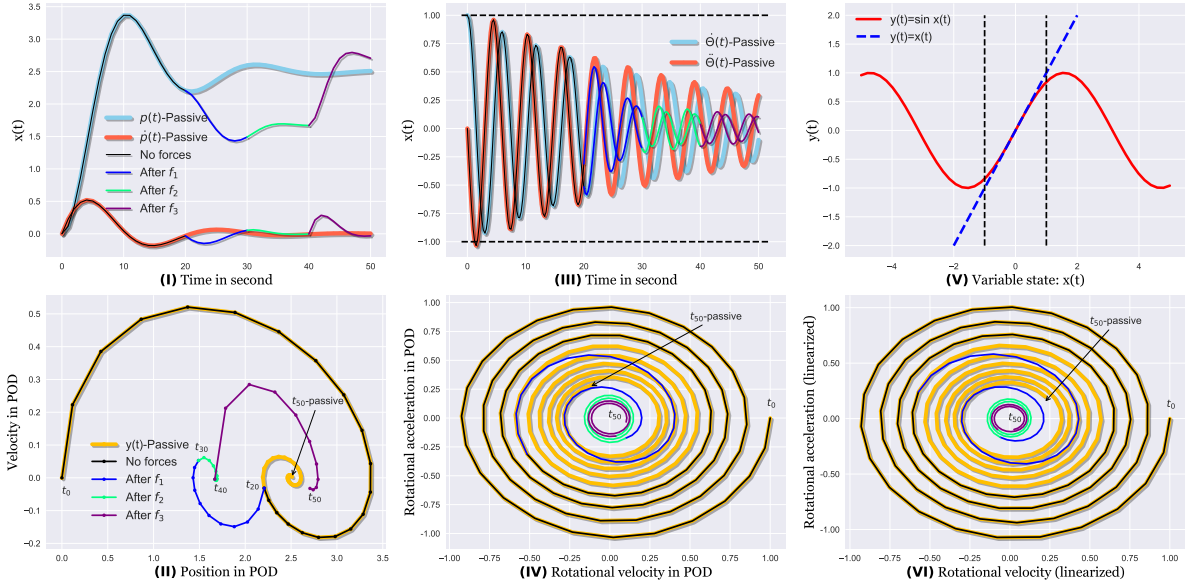
The following forces have been picked to investigate the Mass-Spring-Damper system as a "Linear-Time-Variant" system in the POD framework with the initial input vector,  $u(t_0) = [5N, 20Kg, 2N/m, 4Ns/m]$ .

- (i)  $f_1$ : replacement of spring at time  $t_{20}$  with a higher constant  $\rightarrow k(t_{20}) = 3N/m [k(t_0) \text{ to } k(t_{19}) = 2]$ ,
- (ii)  $f_2$ : concurrent replacement of the following elements at time  $t_{30}$ . (i) mass with a lighter mass  $\rightarrow m(t_{30}) = 10Kg [m(t_0) \text{ to } m(t_{29}) = 20]$  and (ii) damper with a higher damping coefficient  $\rightarrow b(t_{30}) = 6Ns/m [b(t_0) \text{ to } b(t_{29}) = 2]$ ,
- (iii)  $f_3$ : applying a greater force at time  $t_{40} \rightarrow f(t_{40}) = 8N [f(t_0) \text{ to } f(t_{39}) = 5]$ .

According to the tuning of the Mass-Spring-Damper system as a "Linear-Time-Variant" system (equation 9) to the POD framework and applied forces, Fig. 5 (I) displays the system's variables  $x(t)$  (position and velocity) for 50 seconds. The passive trajectory (system variables with initial input vector  $u(t_0)$ ) and deviations (active trajectory) due to  $f_1, f_2$ , and  $f_3$  have been depicted if the chosen forces are applied together. In Fig. 5 (II), trajectories (passive and active) of the tuned system into the POD framework and built on its KPIs (output  $y(t) = x(t)$ ) have been depicted analogously; *with the difference that the whole system viewed as an object in this framework that is being pulled and pushed by generated forces.*

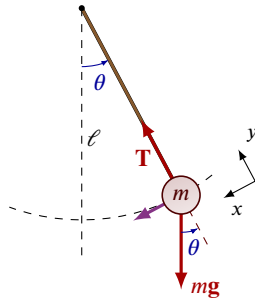
The initial passive trajectory is depicted for the period from  $t_0$  until the moment that the first force arrives ( $f_1$  at  $t_{20}$ ). In linear systems like Mass-Spring-Damper, after  $f_1$  takes effect, a new passive trajectory is formed that takes into account both  $f_1$  and the initial passive impacts with itself starting at time  $t_{20}$  (input vector  $u(t_0)$  and  $u(t_{20})$ ). Likewise, this rule holds for later times and possible incoming forces applied to the system. The vector sum of applied forces is well established in such systems ( $\sum_{i=1}^{i=p} f_i = f_1 + f_2 + \dots + f_i$ ), making it feasible for applied forces to be aggregated (as in classical physics). The equalized KPI-oriented concepts (variation, growth, and fluctuation) to the Kinematic physical notations (displacement, velocity, and acceleration) in section 3.1 is employed directly for system analyses (taking operational and strategic decisions). If either the state or the output equations-or both-are nonlinear, the vector sum of applied forces is invalid ( $\sum_{i=1}^{i=p} f_i \neq f_1 + f_2 + \dots + f_i$ ).

The following example illustrates such systems and potential solutions to deal with this issue.



**Figure 5:** Mass-Spring-Damper and Pendulum systems tuning into the Physics of Decision framework and analyses

The  $u(t)$  argument in equation 9 to the functions can be dropped if the system is unforced (*i.e.*, it has no external force). A classic “nonlinear” system is a simple unforced pendulum shown in Fig. 6.



**Figure 6:** An unforced pendulum system

Equation 10 presents the pendulum motion equation.

$$m\ell^2\ddot{\theta}(t) = -m\ell g \sin \theta(t) - k\ell\dot{\theta}(t) \quad (10)$$

where:

- $g$  is the gravitational acceleration,
- $k$  is coefficient of friction at the pivot point,
- $\ell$  is the radius of the pendulum (to the center of gravity of the mass  $m$ ).
- $\theta(t)$  is the angle of the pendulum with respect to the direction of gravity,
- $m$  is the mass of the pendulum (pendulum rod’s mass is assumed to be zero),

Equation 11 shows the state equation of the pendulum.

$$\begin{aligned} \dot{x}_1(t) &= x_2(t), \\ \dot{x}_2(t) &= -\frac{g}{\ell} \sin x_1(t) - \frac{k}{m\ell} x_2(t). \end{aligned} \quad (11)$$

where:

- $x_1(t) = \theta(t)$  is the angle of the pendulum,
- $x_2(t) = \dot{x}_1(t)$  is the pendulum’s rotational velocity,
- $\dot{x}_2(t) = \ddot{x}_1(t)$  is the rotational acceleration.

Finally, equation 12 presents the general form of the pendulum state and output equations (11).

$$\begin{aligned} \dot{x}(t) &= \begin{bmatrix} \dot{x}_1(t) \\ \dot{x}_2(t) \end{bmatrix} = \Phi(x(t)) = \begin{bmatrix} x_2(t) \\ -\frac{g}{\ell} \sin x_1(t) - \frac{k}{m\ell} x_2(t) \end{bmatrix} \\ y(t) &= \begin{bmatrix} 1 & 1 \end{bmatrix} \begin{bmatrix} \dot{x}_1(t) \\ \dot{x}_2(t) \end{bmatrix} \end{aligned} \quad (12)$$

Similar to the Mass-Spring-Damper system example, Fig. 5 (III) and (IV), with the same equivalent legends in (I) and (II) respectively, present the state variables (rotational velocity and rotational acceleration) and output vector  $y(t)$  in POD together with the initial input vector  $u(t) = [g(t), k(t), \ell(t), \theta(t), m(t)] = [u_1(t), u_2(t), u_3(t), u_4(t), u_5(t)] = [10m/s^2, 2, 8m, 30^\circ, 5Kg]$  at  $t_0$  and the deviations in accordance with the following forces at  $t_{20}, t_{30}$  and  $t_{40}$ .

- $f_1: k(t_{20}) = 10 [k(t_0) \text{ to } k(t_{19}) = 2]$ ,
- $f_2$ : including  $f_{21}: \ell(t_{30}) = 6m [\ell(t_0) \text{ to } \ell(t_{29}) = 8]$  and  $f_{22}: m(t_{30}) = 30Kg [m(t_0) \text{ to } m(t_{29}) = 5]$ ,
- $f_3: g(t_{40}) = 8m/s^2 [g(t_0) \text{ to } g(t_{39}) = 10]$ .

The presented “continuous time-variant” pendulum system is no longer linear due to the term  $\sin x_1(t)$  in equation 11; hence such nonlinear systems don’t obey the forces aggregation rule analogous to linear systems.

Since the  $x(t)$  in Fig. 5 (III) alters almost in the  $(-1, +1)$  interval (shown in black dashed lines) and the term  $\sin x_1(t)$  is almost the same as  $x_1(t)$  on this interval, so  $\sin x_1(t)$  in the state equation is estimated with  $x_1(t)$  (Fig. 4 (V)). **This estimation makes the state equation linear; thus, the forces aggregation works like linear systems.** Fig. 5 (VI) depicts the linearized pendulum system, which almost resembles the original nonlinear trajectory shown in Fig. 5 (IV) (According to Fig. 5 (V), as the pendulum moves away from the point  $(0, 0)$ , the absolute value of the approximation,  $x(t)$ , is greater than the actual value,  $\sin x(t)$ ; thus, the movement of the pendulum is slightly faster than the original movement, and this shift is evident when comparing the Fig. 5 (IV) and Fig. 5 (VI)).

The system equilibrium or stationary points are those when the following condition is satisfied. The applied forces are neutralized by each other ( $F_{net} = 0$ ), so that  $\dot{x} = 0$  [13]. The equilibrium points of a pendulum are those that satisfy  $[x_1(t), x_2(t)] = [n\pi, 0]$  for  $n \in \mathbb{Z}$  (same for the Mass-Spring-Damper system when  $kp(t) + b\dot{p}(t) = f(t)$ ) in Fig. 4.

Real-world systems are almost entirely nonlinear; therefore, applying the state-space model to them is not straightforward as it is for LTI systems (equation 3). It turns out that many nonlinear systems have characteristics that are strikingly comparable to those of their linear counterparts. In addition, various methods make it possible to use linear analysis on nonlinear systems. The linearization of nonlinear systems is a potent method in nonlinear analysis. In dynamics analysis, particularly close to equilibrium points, linearization is used to represent a nonlinear system of ODEs as a linear system [20, 21]. It is possible to employ linearized systems to construct dynamical system controllers that benefit from cutting-edge linear controller approaches to comprehend the local behavior of dynamical systems, notably at equilibrium points [20].

The Taylor series of dynamical ODEs is one of the most widely used linearization methods. The general 1st order structure of the Taylor series is recalled by equation 13 to linearize the function  $\Phi$  in equation 1 around point  $(x, u)$  [21].

$$\dot{x}(t) = \Phi(x, u) \approx \Phi(\bar{x}, \bar{u}) + \left. \frac{\partial \Phi}{\partial x} \right|_{\bar{x}, \bar{u}} (x - \bar{x}) + \left. \frac{\partial \Phi}{\partial u} \right|_{\bar{x}, \bar{u}} (u - \bar{u}) \quad (13)$$

If the values of  $x$  and  $u$  are chosen in steady state conditions (at equilibrium points) then  $\Phi(\bar{x}, \bar{u}) = 0$  because the derivative term  $\partial \Phi / \partial u = 0$  in steady states.

The POD framework is supplemented with the linearization method to assess the aggregation of individual decisions (applied forces) in nonlinear case studies to control the system trajectory in the performance space  $\mathcal{Y}$ . To state it plainly, **in nonlinear systems, aggregation of taken individual decisions, where the decisions are viewed as forces**

**that move the system in its performance space, will not necessarily be the same as if they were taken together.** This issue mainly arises from cascading effects and subsequently delayed implications in nonlinear systems (see Figure 1). Section 4 goes into further detail about this issue and applies the introduced solution to an extended example.

## 4. Experiments and results

The main experiments are centered on the significance of the force-based paradigm in the POD framework as a DSS on industrial and strategic simulation models. The presented POD framework in this study constrains the experiments by requiring differential equation models of the systems being studied in this form so that the equivalent state-space model to the system’s ordinary differential equations (ODEs) could form (however, the other POD studies have been implemented through different simulation approaches such as agent based modeling (ABM) [15] and discrete event modeling (DEM) [12] too). The most prevalent form of ODE modeling is the System Dynamics approach (compartmental models), consisting of compartments, flows, causal loop diagrams (CLD), and system parameters. The POD answers the following critical business question in nonlinear systems that the System Dynamics approach, due to nonlinearity, can not address.

“What if the system manager decides to take several individual decisions which cost the system less (easier path or farther from intended danger zones, or both, in the system’s performance space (Fig. 2)) if they were taken together and then aggregate them to evaluate the system performance?”

The presented work and supplied procedures assess two well-known *nonlinear* models: a simple illustrative model to apply the linearization and forces aggregation, then an industrial and complicated model, both implemented with the System Dynamics approach to address this issue.

### 4.1. Bass diffusion system dynamics model

The dynamic model of a basic Bass Diffusion system implemented with ODEs is used to illustrate the state-space method and linearization procedures in nonlinear dynamical systems where the state-space representation in equation 3 cannot be directly applied [22].

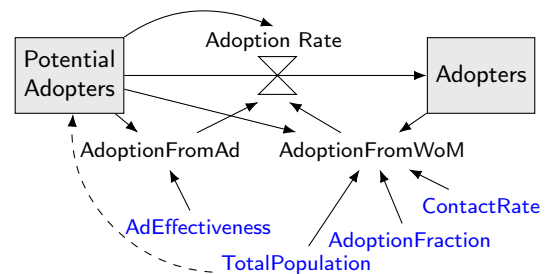
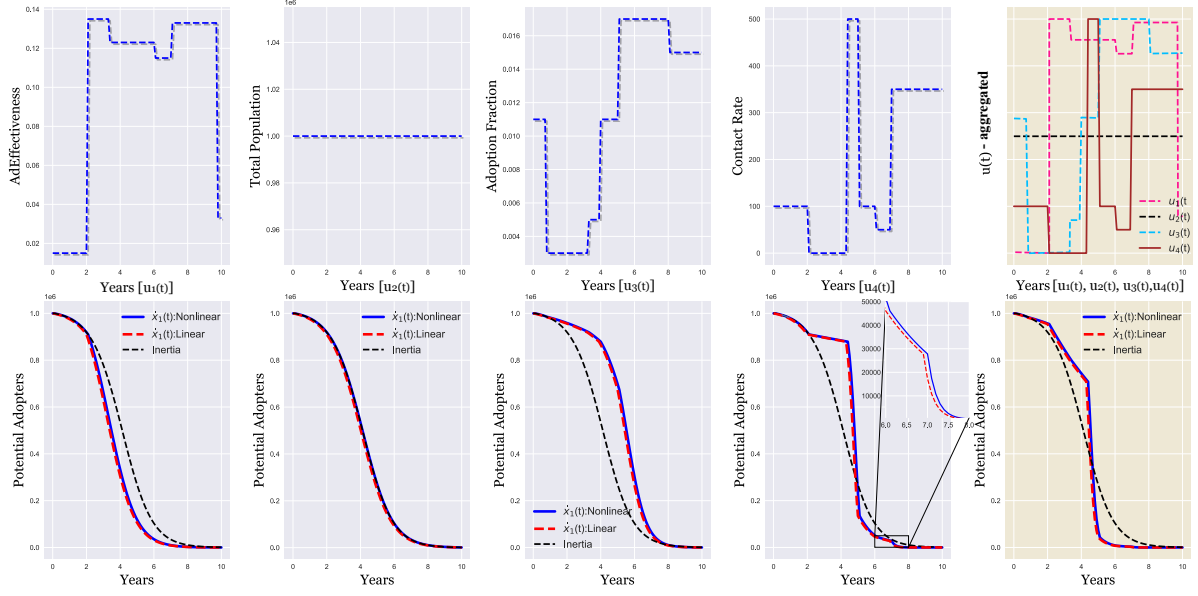


Figure 7: Basic Bass Diffusion system dynamic model

This model consists of a simple differential equation describing how “new products are adopted in a population.”





**Figure 8:** Individual inputs' variations (applied forces) and their aggregations vs. output (potential adopters)

The model presents a rationale for how current and potential adopters of a new product interact. The premise of the strategy is that potential adopters might become new customers through advertisement or word-of-mouth (WoM) mechanisms among potential and existing consumers. The Adoption Rate is the accumulation of these two ways of adoption (Advertise and WoM) at any given moment [23]. The potential and current Adopters variations are considered as state variables  $x_1(t)$  and  $x_2(t)$ , respectively. The population individuals are either customers or potential customers, meaning that  $x_1 + x_2 = \text{Total population}$  ( $u_2$  latter) and, consequently,  $\dot{x}_1 = -\dot{x}_2$ . Equation 14 presents the *state* and *output* equations of the Bass Diffusion presented in Fig. 7.

$$\begin{aligned}
 \dot{x}_1(t) &= -(\text{AdoptionFromAd} + \text{AdoptionFromWoM}) \\
 \dot{x}_1(t) &= \Phi(x(t), u(t)) = -x_1 u_1 - x_1 x_2 u_3 u_4 / u_2 \\
 \dot{x}(t) &= \begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \end{bmatrix} = \begin{bmatrix} -x_1(u_1 + u_3 u_4) + x_1^2 u_3 u_4 / u_2 \\ x_1(u_1 + u_3 u_4) - x_1^2 u_3 u_4 / u_2 \end{bmatrix} \\
 y(t) &= \begin{bmatrix} 1 & 0 \end{bmatrix} \begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \end{bmatrix}
 \end{aligned} \tag{14}$$

where:

- $u_1(t)$  is AdEffectiveness: Advertisement influence coefficient on Adoption Rate,
- $u_2(t)$  is Total Population: Total adopters and potential adopters,
- $u_3(t)$  is Adoption Fraction: The imitation rate of potential adopters from current adopters,
- $u_4(t)$  is Contact Rate: Contact rate between potential adopters and current adopters,
- $u(t) = [u_1(t), u_2(t), u_3(t), u_4(t)]$ .

The state equation of the Bass Diffusion model in equation 14 is nonlinear (due to the term  $x_1 x_2$  before arrangement, later  $x_1^2$ ) that requires linearization. The presented Jacobian Matrix in equation 15 linearizes this state variable concerning input vector  $u(t)$  to be in the “continuous time-variant” state-space form (equation 9). Note that  $\partial x / \partial x_1 = -u_1 - u_3 u_4 + 2x_1 u_3 u_4 / u_2$ .

$$J(\bar{x}) = \begin{bmatrix} \frac{\partial x}{\partial u_1} & \frac{\partial x}{\partial u_2} & \frac{\partial x}{\partial u_3} & \frac{\partial x}{\partial u_4} \end{bmatrix} = [-x_1, -u_3 u_4 x_1^2 / u_2^2, -u_4 x_1 + u_4 x_1^2 / u_2, -u_3 x_1 + u_3 x_1^2 / u_2] \tag{15}$$

Fig. 8 shows the consequences of changing inputs individually ( $u_1, u_2, u_3$ , and  $u_4$  on top) on the Potential Adopters as intended output in linear and nonlinear forms at the bottom. The final plot (the rightmost at the bottom) demonstrates the impact of aggregated input changes ( $u_1, u_2, u_3$ , and  $u_4$ ) on the output in both linear and nonlinear situations.

According to the zoomed portion of the “Contact Rate” variation plot in Fig. 8, the comparison between linearized and nonlinear versions indicates the following conclusions.

- The 1st order linearization from the Taylor series is sufficient ( $x_1^2 \approx 0$ ) to transform the nonlinear state equation 14 to continuous time-variant form (equations 9) and consequently compute the effects of input variations on the output for the basic Bass Diffusion model, yet; highly oscillated models require higher orders.
- The forces have immediate impacts on the output in the nonlinear form; however, there are considerable behind-time impacts in the linearized versions. For instance, consider the variation of the Contact Rate from 50 to 350 contacts per person starting at the beginning of the 7th year."

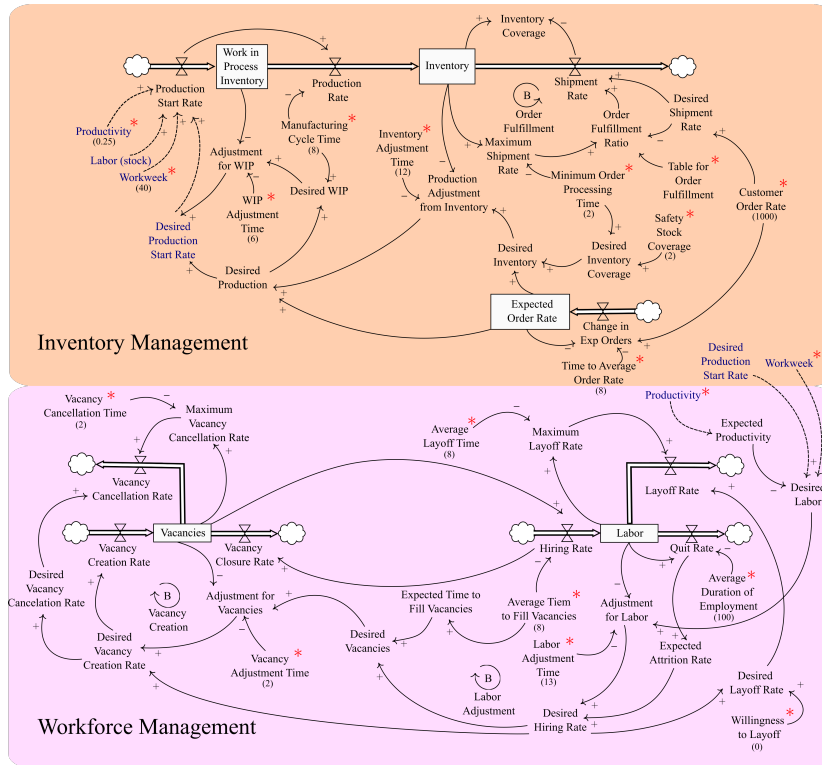


Figure 9: System Dynamic model of inventory-workforce management provided in [24]

(iii) The forces aggregation in the linear version closely resembles the application of forces together in the nonlinear form (the depicted plots in the rightmost at the bottom of Fig. 8).

#### 4.2. Inventory-workforce system dynamics model

This section presents a more industrial, highly oscillated, and continuous time-variant system dynamic approach of an inventory-workforce model, which is entirely nonlinear in nature. The study employs the linearization approach to compute the linear counterpart of the nonlinear inventory-workforce system at the time of input variations (applied forces), and it calculates the KPIs variation, growth, and fluctuation (section 3.1), which are connected to the physical notations of displacement, velocity, and acceleration, respectively [13].

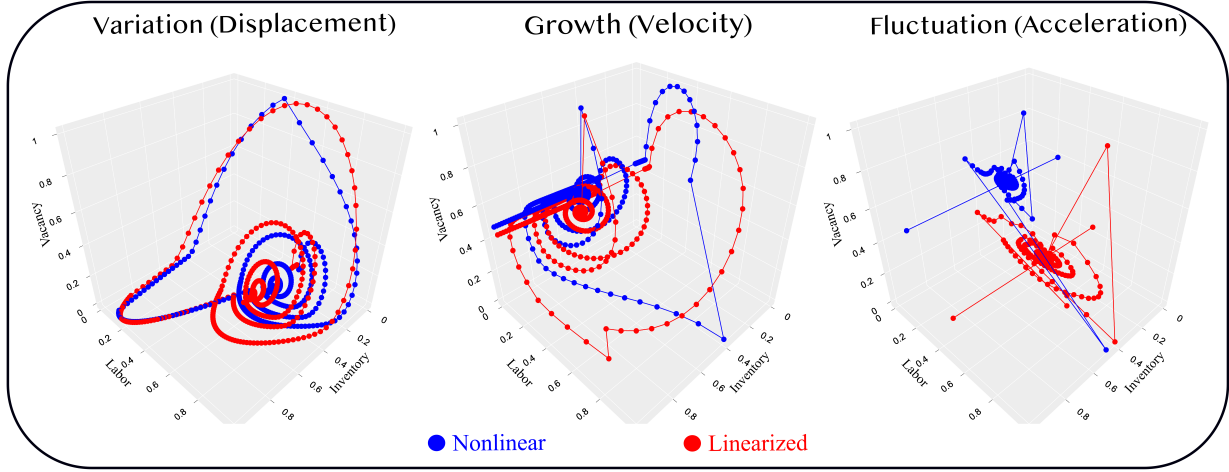
The inventory-workforce model illustrates how production scheduling and employment regulations interact, potentially causing inconsistency in responding to consumer demand effectively.

The model makes some significant simplifying assumptions. Order backlog is ignored and customer orders are exogenous. The inventory-workforce model emphasizes labor's importance as a production factor. When the workforce model and the inventory model are combined, production starts to adapt with a delay to the targeted start time. The inventory level with the connected workforce model decreases further after the demand shock (high order from customers) than the inventory model without it.

The inventory model structure and the equations between its parameters, stocks, flows, and causal loop diagrams (CLD) are thoroughly described in chapter 18 of the “*Business Dynamics*” book [24]. Chapter 19 discusses the workforce model and its link to the inventory model. The behavior of the coupled model, including oscillations and their sources, is also explored [24].

This study avoids an in-depth repetition of how the models operate and instead provides a schematic representation of the structure of the models and the relationships between the parameters, stocks, and flows as presented in Fig. 9. Dashed arrows within the diagram denote the shared parameters of the two interconnected models. Detailed explanations of the dynamics and mechanisms of the inventory-workforce model are covered in the appendix A of this paper. This approach allows us to focus primarily on applying the force-inspired paradigm to time-dependent nonlinear models (e.g., system dynamic models) rather than delving into the intricacies of the models themselves. The goal of this research is to propose a decision-making tool that leverages these models.

The inventory-workforce model is tuned to the POD framework presented in section 3 as follows. The POD framework considers the parameters in Fig. 9 with just output arrows (this property is present in 17 parameters) as inputs in the space  $\mathcal{U}$ . Fig. 9 depicts the model's inputs with an asterisk symbol (\*). The remaining components, such as flows (e.g., “Shipment Rate”), stocks (e.g., “Labor”), and hybrid variables (e.g., “Desired Vacancies”) are outputs in



**Figure 10:** Nonlinear and linearized trajectories for variation, growth, and fluctuation of the selected KPIs

the space  $\mathcal{Y}$ . Input parameters are those that the simulation modeler is able to assign values and alter (\*). On the other hand, hybrid variables can be utilized as input from a different point of view. The distinction between inputs and outputs is clearly based on the system management's perspective; it is just a way to separate inputs from outputs inside the POD framework. Following the mapping management concepts onto physical notations, then the linearization approach through the Taylor series and forces aggregation in the inventory-workforce model are explored.

The experiments begin a passive trajectory with the force of 1000 "Customer Order," which triggers the entire system (other input parameters). The input parameters in Fig. 9 with the (\*) symbol have been initialized to the values in parentheses to form the passive trajectory. The following significant experiment contrasts individual decisions and their aggregation in linear (or linearized) versus nonlinear circumstances. This evaluation enables mitigating risks brought on by unforeseen outcomes brought on by shifts in system performance by comparing the tentative decision-making that is not the same as definitive decision-making due to the system's nonlinearity. The assertion of the significant experiment is backed by the forces having the properties listed below. (i) Selected inputs to vary (force creation) belong to both the inventory and workforce sectors; (ii) Forces have indirect impacts on the concerned KPIs (Fig. 9: to be checked through the path from input to the KPIs), and (iii) The applied forces are studied for relatively long periods to take into consideration the simultaneous influence of the interacting forces on the KPIs (not only a brief shock to the system).

- (i)  $f_1$ : The "Safety Stock Coverage" climbed from 2 to 4 products over the timeframe (50,70): from the inventory sector,
- (ii)  $f_2$ : The "Productivity" descends from 40% to 25% over a (100,150) timeframe: from workforce sector,

- (iii)  $f_3$ : The "Average Time to Fill Vacancies" descends from 10 to 8 over a (120,180) timeframe: from the workforce sector.

Fig. 10 depicts the nonlinear and linearized trajectories for inventory, labor, and vacancies on axes, respectively, for variation, growth, and fluctuation. A pre-process *fit\_transform* has been performed on the trajectories to scale the KPIs values to be able to have a vis-a-vis study. This procedure allows simultaneous investigation of trajectories despite different units of indicators (e.g., the Inventory unit is "product" while the Labor unit is "person"). First, the *MinMaxScale* estimator scales and translates each KPI individually such that it is in the given range (in this experiment, between zero and one instead of the actual values of KPIs); then, the *fit* method calculates the mean and variance of each present feature in the data. The *transform* method transforms all the KPIs through respective mean and variance. Fig. 10 aims to illustrate the main objective of the study, which is to highlight performance shifts in nonlinear and linearized systems and emphasize the essential need to employ the decision aggregation method through linearization and the POD framework. By viewing complex systems holistically (e.g., a moving object in a multidimensional space), these methods provide insights into system complexities and facilitate affordable decision-making. This is achieved by drawing analogies between management concepts and physical notation, as well as through kinematic analysis.

Essentially, the coupled inventory-workforce model has been designed to harness system dynamicity through parameter configuration and tuning, thereby fulfilling system demands over time. By adjusting these parameters and employing regularization, the model applies forces that gradually neutralize as time progresses, akin to the behavior observed in the tail of the displacement plot in Fig. 10. Similarly, in physics, the "restoring force" is a force that acts to bring a body to its equilibrium position. The restoring force is always directed back toward the system equilibrium position

that depends only on the mass or particle position (the same as the system viewed as an object in this study) [25].

In nonlinear systems like the inventory-workforce model, variations in several parameters simultaneously, as opposed to adjustments to individual parameters, can result in significantly different outcomes. This inherent nonlinear dependency is pivotal as it mirrors real-world complexities where multiple aspects of a system change concurrently. Linearization is thus employed to simplify these interactions, enhancing manageability and improving predictability under specific conditions. This approach proves particularly effective when deviations from a set equilibrium are slight and the system's responses to these changes remain relatively proportional. By reducing complexity, linearization allows for a more straightforward analysis of how changes in system parameters impact overall behavior, making it easier to anticipate and mitigate potential disruptions in system performance.

The applied forces,  $f_1$ ,  $f_2$ , and  $f_3$ , have been chosen so that the linearization significantly estimates the nonlinear system. In other words, the applied forces are not drastic (e.g., raising the "safety stock coverage" from 2 to 4 units), meaning that the linear system is only an approximation that holds for slight deviations from equilibrium. This keynote highlights the inefficiency of decision aggregation in linearization technique under crisis-like circumstances with drastic variations in system parameters (applied forces to the system). Despite the resemblance between the displacement plots, which are primarily associated with operational decision-making, the notable differences in the velocity and acceleration (fluctuation) plots highlight their critical roles in strategic decision-making. In particular, the acceleration plot illustrates how strongly or weakly the KPIs fluctuate in the system's performance space (see section 3.1) and underscores the importance of linearization in the decision aggregation process in nonlinear systems to better predict system performance. These distinctions between nonlinear and linearized systems emphasize the importance of the POD framework for a better alignment between operational and strategic decisions, suggesting the importance of integrating both perspectives to account for differing system behaviors when formulating comprehensive strategic plans. Overall, linearized solutions are recommended when the system parameters are stable or exhibit only minor fluctuations. In scenarios where parameters undergo significant or rapid changes, the linear model may fail to capture the true dynamics of the system, leading to suboptimal decisions.

## 5. Conclusions and perspectives

This study, on top of the "Physics of Decision" principles [5, 9], first tunes the simulation models to well-known physical simulation models called "state-space representation" and then addresses the aggregation of the decisions in nonlinear systems. "The importance of decision aggregation is that taking numerous decisions at once is cumbersome, overpriced, and unfuturistic; in contrast, taking the same

decisions independently and then aggregating them is more advantageous from a financial and futuristic standpoint. This approach assists decision-makers in (i) exploring a variety of strategies and taking the best ones, (ii) allocating resources firmly, and (iii) driving the system toward its objectives through constant avoidance of determined risky zones and proximity to safe areas (targeted objectives) in the system performance space built on systems indicators, that allows them to be more readily in seizing opportunities and avoiding risks.

The work's significance has been examined in the experiment section, first with an outdated but illustrative example from the Bass Diffusion model, after with a nonlinear model of an inventory-workforce system developed with the System Dynamic modeling. Our research employs a coupled inventory-workforce model, focusing on linearization to aggregate decisions and manage perturbations effectively, as illustrated in Fig. 10. This approach highlights the nonlinear nature of real-world systems where simultaneous parameter variations can significantly alter outcomes, making linearization a useful tool for simplifying complex interactions and enhancing predictability when deviations from equilibrium are minimal. However, linearization's effectiveness is limited under conditions of drastic or rapid changes, where it may fail to capture true system dynamics, leading to potentially suboptimal decisions. This emphasizes the need for cautious application of linearization, particularly in stable environments to effectively support decision-making processes.

Three primary places are listed below to develop the presented work and further research.

- (i) Model: The presented work deals with System Dynamic models implemented with Ordinary Differential Equations (ODEs); however, including alternative simulation approaches such as agent-based modeling (ABM), discrete event modeling (DEM), and others is an essential development aspect,
- (ii) Visualization: although the performance space is not limited to three dimensions, it is impossible to examine more than three KPIs simultaneously. Virtual reality (VR) technologies could map and analyze how system inputs affected the performance trajectory for several KPIs at once to look at how they are related, and
- (iii) Complexity: A comprehensive sensitivity analysis of the system's inputs and outputs could help in determining the impact pattern of inputs and outputs (such as linear, wave-forms (sine, square, triangular, ramp), etc.) to examine the performance trajectory of both with and without them as well as to decompose the performance trajectory into multiple trajectories with distinct patterns to have a rigorous linearization.

## CRedit authorship contribution statement

**Nafe Moradkhani:** Conceptualization, Data curation, Formal analysis, Investigation, Methodology, Writing - Original draft preparation. **Frederick Benaben:** Supervision, Conceptualization, Investigation, Writing - review & editing. **Benoit Montreuil:** Supervision, Writing - review & editing. **Matthieu Lauras:** Supervision, Investigation, Writing - review & editing. **Thibaut Cerabona:** Validation, Visualization, Writing - review & editing. **Clara LeDuff:** Validation, Visualization. **Louis Faugere:** Validation, Writing - review & editing. **Julien Jeany:** Writing - review & editing.

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## A. Inventory-Workforce Model in section 4.2

The Inventory-Workforce model, as presented in this paper, offers a fundamental understanding of how inventory management and workforce dynamics interact within a business environment to ensure effective responses to consumer demand. This model intricately links production rates to inventory levels through adaptive feedback loops. For example, when inventory levels fall below the desired threshold, the model triggers an increase in production rates, whereas excess inventory leads to a reduction in production activities. While our paper outlines the general principles and basic mechanics of these interactions, including how labor factors such as availability, productivity, and turnover influence overall inventory management, the complexities and full mathematical formulations of these relationships are beyond the scope of this discussion. For those readers seeking a deeper understanding of the underlying equations and advanced simulations that describe the dynamic interplay between these elements, Chapter 19 of the "Business Dynamics" book [24] remains an invaluable resource. This reference provides extensive insights into the systematic adjustments and analytical framework required to fully grasp and implement the Inventory-Workforce model effectively.

### A.1. Inventory Management

The inventory management part of the model focuses on the flow of products through different stages of production and inventory.

1. **Work in Process Inventory (WIP):** Represents the intermediate stage where items are being processed. The production start rate influences the production rate, which subsequently affects the WIP inventory. The adjustment for WIP ensures that production rates align with the desired WIP levels, maintaining a balance between work in process and production output.

2. **Production Rate:** Determined by the available labor, productivity, and workweek hours, the production rate is a crucial factor in managing the flow from WIP to finished inventory. Higher productivity or an increase in labor can boost the production rate, accelerating the transition from WIP to finished goods. It directly influences how quickly inventory is replenished to meet customer demand.
3. **Inventory:** This is the stock of finished products ready to meet customer orders. Inventory adjustment is performed based on the difference between desired inventory and actual inventory levels. This adjustment considers factors such as the inventory adjustment time and desired inventory levels to ensure optimal stock levels are maintained. Inventory coverage, the ratio of actual inventory to desired inventory, is a key metric in determining stock adequacy.
4. **Order Fulfillment:** Customer orders drive the shipment rate, which depletes the inventory. The model assumes a constant customer order rate, influencing the desired shipment and order fulfillment ratio. Order fulfillment is essential for meeting customer demand and maintaining satisfaction. The shipment rate must be aligned with both current inventory levels and anticipated customer orders.
5. **Production Adjustment:** Adjustments are made to the production rate based on inventory levels and desired production rates, ensuring that the inventory remains at optimal levels. This involves modifying production in response to changes in inventory to avoid overproduction or stockouts. Factors such as the manufacturing cycle time and the desired production start rate play a crucial role in these adjustments.
6. **Inventory Coverage:** Represents the ratio of actual inventory to desired inventory. It is a critical measure for determining whether the inventory is sufficient to meet expected demand. Inventory coverage helps in making informed decisions about production rates and adjustments needed to maintain stock levels.
7. **Expected Order Rate and Change in Expected Orders:** These parameters forecast future demand and adjust production rates accordingly. They are essential for planning and ensuring that production schedules align with anticipated changes in order rates. By predicting future demand, the model can preemptively adjust production to meet upcoming needs.
8. **Production Start Rate:** Influences the rate at which items enter the production process. It is adjusted based on desired production levels and current inventory status. Ensuring an appropriate production start rate is vital for maintaining a smooth flow from WIP to finished goods without creating bottlenecks.

The inventory management system dynamics model describes how production and inventory levels are regulated within a business environment. It emphasizes the importance of aligning production rates with inventory needs to effectively meet customer demand. The model incorporates feedback loops where inventory levels influence production adjustments and vice versa. For instance, a lower-than-desired inventory level triggers an increase in the production rate, while a higher-than-desired inventory might reduce production. By simulating these interactions, the model provides insights into maintaining optimal inventory levels, minimizing costs, and ensuring timely order fulfillment. It also helps in understanding how various factors such as labor availability, productivity, customer orders, and shipment rates interact to affect overall inventory management.

## A.2. Workforce Management

The workforce management part of the model focuses on the dynamics of hiring, layoffs, and labor productivity.

1. **Vacancies:** Represents the number of open positions within the organization that need to be filled. The vacancy creation rate determines how many new vacancies are generated based on business needs, while the vacancy cancellation rate closes vacancies that are no longer required. Adjustments for vacancies ensure that the number of vacancies aligns with the desired levels.
2. **Vacancy Creation Rate:** Dictated by the business needs, this rate determines how many new positions are created. An increase in production demand or expansion of business operations can lead to a higher vacancy creation rate.
3. **Vacancy Cancellation Rate:** Represents the rate at which open positions are canceled when they are no longer needed. This can happen due to changes in business strategy, automation, or a reduction in demand.
4. **Hiring Rate:** Determines the rate at which vacancies are filled by hiring new employees. The hiring rate depends on the desired vacancies and the expected time to fill vacancies. It directly influences the labor available for production.
5. **Labor:** Represents the total workforce available for production. It is influenced by the hiring rate, layoff rate, and quit rate. Adjustments to labor levels are made based on the desired labor, which is determined by the production needs. Productivity levels and the standard workweek also affect the effective labor contributing to production.
6. **Layoff Rate:** Indicates the rate at which employees are laid off. The layoff rate is influenced by factors such as the desired layoff rate, average layoff time, and the current labor levels. Layoffs are used to adjust the

workforce size to align with reduced production needs or cost-cutting measures.

7. **Quit Rate:** Represents the rate at which employees voluntarily leave the organization. It affects the overall labor levels and can impact production if not balanced by an adequate hiring rate.
8. **Productivity:** Measures the efficiency of the workforce. Higher productivity means more output per labor hour, directly influencing the production rate. Productivity adjustments ensure that the workforce efficiency aligns with expected productivity levels.
9. **Adjustment for Vacancies:** Ensures that the number of vacancies aligns with desired levels. It considers the vacancy creation rate and vacancy adjustment time to maintain an optimal balance between open positions and business needs.
10. **Adjustment for Labor:** Adjusts the workforce size based on the difference between desired labor and actual labor levels. This adjustment considers the labor adjustment time to smoothly transition workforce levels without causing disruptions.
11. **Willingness to Lay Off:** Represents the organization's policy or inclination towards laying off employees. A higher willingness to lay off can result in a higher layoff rate when labor adjustments are needed.
12. **Average Duration of Employment:** Indicates the average time employees stay with the organization. It affects the quit rate and can be used to forecast labor stability and turnover.
13. **Average Layoff Time:** The typical time taken to process layoffs. This parameter influences the layoff rate and how quickly labor adjustments can be made in response to changing production needs.
14. **Average Time to Fill Vacancies:** The average time required to hire new employees for open positions. It affects the hiring rate and the ability to quickly respond to increases in vacancy creation rate.
15. **Vacancy Adjustment Time:** The time taken to adjust the number of vacancies to align with desired levels. This parameter helps in managing the vacancy creation and cancellation rates effectively.
16. **Vacancy Cancellation Time:** The average time taken to cancel a vacancy that is no longer needed. It influences the vacancy cancellation rate and helps maintain the balance between open positions and actual hiring needs.

no longer required. The hiring rate fills these vacancies, contributing to the labor pool. The labor pool is adjusted through hiring, layoffs, and voluntary quits to maintain optimal workforce levels for production. Productivity and workweek parameters further refine the effective labor available for production, ensuring that the production rate meets inventory and customer order requirements. Feedback loops ensure that adjustments in one area (e.g., labor) affect other areas (e.g., production rate), providing a dynamic and interconnected model of workforce management.

The workforce management system dynamics model illustrates how hiring, layoffs, and productivity adjustments influence the available labor for production. Vacancies are created based on business needs and are canceled when