An exact two-phase approach to re-optimize tours in home care planning

Clea Martinez, Marie-Laure Espinouse, Maria Di Mascolo

An exact two-phase approach to re-optimize tours in home care planning

Clea Martinez[1], Marie-Laure Espinouse[2], Maria Di Mascolo[2]

Abstract

With the increase in demand, home care agencies must find efficient ways to schedule and route their staff. Unfortunately, the home care sector is never completely stable, the pool of patients constantly evolves, and the staff is subject to a high turnover. Therefore, home care agencies need to regularly update the schedules and to re-plan the visits of careworkers to patients. In this article, we present a method to re-assign the careworkers and re-design their tours whenever a schedule becomes obsolete due to the variations within the pool of patients or the staff. The originality of this work lies in the fact that we study the problem at the strategic level, with close-to-reality constraints. We analyze the impact of three different optimization criteria on the composition of the tours. We tested our algorithm on adapted instances from the literature and on instances extracted from real data provided by a home care agency in France, with up to 15 careworkers, 92 patients, and 337 visits over a week.
Keywords: OR in health services, home care, re-assignment, decomposition algorithm

1. Introduction

Home Care (HC) enables fragile people to stay at home even if they are sick or need assistance in their daily life. It may include medical care, skilled services,

[1]Mines-Télécom Institute Albi-Carmaux School, France
[2]Univ. Grenoble Alpes, CNRS, Grenoble INP[3], G-SCOP, 38000 Grenoble, France
[3]Institute of Engineering Univ. Grenoble Alpes

or social services. HC agencies are often preferred to traditional nursing facilities because they are less expensive and often more convenient for the patients (Heath, 2017). Consequently, as western populations age, demand has recently skyrocketed and is expected to keep increasing in the years to come (Rogers, 2018). The sector's growth makes the scheduling task even more complex and time-consuming. As a consequence, it has become necessary to automatize the planning process, which was formerly entrusted to experimented workers of the HC agencies.

Once a schedule is built, numerous contingencies inherent to the health sector disturb the decisions taken at a strategic level. Most common perturbations come from the variations within the pool of patients and careworkers. They have a significant impact on the execution of the schedules, so HC agencies must regularly adapt them at the operational level and work at the strategic level to ensure good performance in the long term. In this article, we focus on the strategic re-optimization of the schedules. Medical and legal constraints must be observed, as well as other constraints emerging from the human dimension of HC, such as continuity of care. We distinguish two types of continuity: human continuity, which consists of assigning the same group of careworkers to the same patients when the planning lasts for several periods (long-term horizon planning); and temporal continuity, which consists of providing periodic services always at the same time. To our knowledge, temporal continuity is treated very little, in the literature.

Most of the time, economic stakes and patients' preferences are brought forward at the expense of the careworkers' well-being. It is to be noted that the HC field is particularly stressful, with difficult working conditions and low recognition. It creates an anxiety-provoking environment that leads to many burn-outs and resignations. The resulting turnover exacerbates the instability of the schedules and significantly impacts service quality. O ering better working conditions is not only a way of limiting turnover but also a convincing argument to hire skilled employees in a highly competitive field.

In this article, we tackle the problem of re-designing careworkers' tours under legal and continuity constraints when an ongoing schedule becomes out-of-date because of variations within the pool of patients and/or careworkers. We take advantage of the particular features of our use case to propose an innovative exact approach based on a modelization of the problem with graphs. The remaining of this article is organized as follows: section 2 offers an overview of the literature related to our problem, and section 3 describes the scheduling and routing problem that we study. In section 4, the method is presented, along with the experimental results in section 5. Finally, we propose conclusions and perspectives for future work in section 6.

2. Literature review

Preamble: Even if HC scheduling problems and HHC (home health care) scheduling problems present some differences, particularly in terms of data (service durations, features of patients, etc.), the distinction between these two fields is not always clear, and the methods developed in the literature to tackle the related problems are very similar. Therefore, we present a state of the art including both HC and HHC studies.

2.1. Home care routing and scheduling problem

The scheduling task, known in the literature as the Home Health Care Routing and Scheduling Problem (HHCRSP), was first studied in (Begur et al., 1997) where the authors consider the weekly problem with a spatial decision support system, which enables them to create and visualize careworkers' tours.
To our knowledge, the first mathematical formulation of the problem was given in (Cheng & Rich, 1998), where it is modeled as a multi-depot vehicle routing problem with time windows and on a single period. Some practical constraints are taken into account, such as part-time and full-time workers, and the objec-

tive is the overtime working hours minimization.

HHCRSP problems are classically formulated as vehicle routing problems, and many variants of VRP problems can be adapted to suit the HC field. For example, in HC agencies, most patients require numerous regular visits, thus the schedule is often built on multiple periods. It can lead to the implementation of patterns or frequency constraints, which are typical of the Periodic Vehicle Routing Problem (PVRP) (Mor & Speranza, 2020).
The incompatibilities between patients and careworkers can be interpreted in a Site-Dependent VRP where not all vehicles can provide all clients (Cordeau & Laporte, 2001). The Consistent Vehicle Routing Problem (Con-VRP) requires that a client is always delivered by a unique vehicle over a horizon of multiple periods. Kovacs et al. (2014) propose a generalization of the problem by noticing that the cost can be largely minimized if more than a single vehicle is allowed per client. They consider the case where every client is delivered by a limited number of vehicles since it enables them to find solutions even when a specific vehicle is not a vailable. This approach seems more realistic notably for HC problems, and it concurs with our definition of human continuity of care. Regarding temporal continuity, it is not a typical constraint in HHCRSP problems. However, time consistency is tackled in other multi-period routing problems applied to health, as in (Tellez et al., 2020) where the authors solve a transportation problem for disabled people. Kovacs et al. (2015) study a trade-off between time consistency and driver consistency
In this paper, since we work on a schedule re-optimization, we can model temporal continuity by keeping the planned starting times, i.e by considering fixed starting times. This modeling choice comes from an actual case of an HC organization, for which the day and the starting time of each service are defined contractually when a patient requires a service for the first time. Since every change in the starting times must then be subject to new bargaining with the patient, it should be avoided when the tours are redesigned on a strategic level.

The literature is extremely rich on HHCRSP problems, and even more for VRP problems, so we chose to focus on studies in the HC/HHC area. Many studies have tackled the HHCRSP problem, as it represents a topical issue not only for HC stakeholders but also for researchers with new scientific and technical obstacles. Most of these works are listed in the recent literature reviews of the field: (Cissé et al., 2017; Fikar & Hirsch, 2017; Grieco et al., 2020; Di Mascolo et al., 2021). In the remainder of this section, we do not intend to be exhaustive, but only to give the reader an overview of the existing work.

As a variant of the vehicle routing problem, the classic HHCRSP is NP-hard, thus a lot of heuristics and meta-heuristics are developed. Some researchers choose to approach the problems through several steps. In (Fikar & Hirsch, 2015) the problem is decomposed into two stages: the identification of potential tours, and then the optimization of the transportation system in its entirety. Grenouilleau et al. (2019) also suggest a two-phase algorithm to tackle the weekly planning problem with overtime costs, non-covered services, and skill requirements. First, feasible routes are created with a large neighborhood search, then a set partitioning model and a constructive heuristic select a set of routes to make a weekly schedule. The experiments are conducted on instances of up to 430 visits. Lahrichi et al. (2022) propose a First-Route-Second-Assign decomposition: a giant tour is obtained during the first phase, and is then split into subtours in a second phase.

Most often, the considered objectives are the classic objectives of VRP problems: minimization of travel times, costs, or distance. We can refer to (Bard et al., 2014), where both travel times and overtime costs are minimized. They propose a MILP formulation and a GRASP algorithm for the weekly problem with legal constraints, time windows, and skills requirements, and apply their method to instances of up to 45 patients.
It is to be noted that particular interest is also being paid to patients' satisfaction. In (Mosquera et al., 2019), patients have preferences over the frequen-

cies, durations, and starting times of the services they request. The services have di erent levels of priority and are hierarchically considered in the objec-tive function, which minimizes the deviation from the preferences expressed by the patients. A greedy algorithm generates an initial solution, which is then improved by a local neighborhood search. Borsani et al. (2006) maximize conti-nuity of care in the weekly HHCRSP, solved with various MILP. Their method was tested on real instances provided by an Italian home care agency with 25 careworkers.

More recently, some articles also consider the satisfaction of the careworkers. In (Decerle et al., 2019), several criteria are studied: the minimization of travel times and the workload balance between sta members. Trautsamwieser & Hirsch (2011) use an aggregation of travel and waiting times of the sta , over-time hours, and penalties caused by overqualified work. Computations were performed on instances of 9 sta members and 203 visits over the week. There are three stakeholders in the home care scheduling problem - the manager of the HC agency the sta members, and the patients - whose interests do not always go hand in hand. That is why Carello et al. (2018) study the trade-o be-tween all three stakeholders with a MILP formulation and a threshold method. Overtime costs are minimized, the maximal utilization rate of careworkers is minimized, and the continuity of care is maximized.

2.2. *Uncertainties, contingencies, and perturbations*

In most articles, all parameters are assumed to be known in advance, and the schedule is built from scratch. However, the HC field brings many uncertainties and/or variability in the data. The turnover of the sta is to be taken into ac-count since it has a big impact on the practical application of a schedule, as well as the high variability of the pool of patients. If a schedule becomes irrelevant because of these perturbations, there is the need to re-schedule the initial plan. Recent articles tend to take into account those contingencies and disruptions which represent a real hindrance for HC agencies and can be handled either at a strategic (long-term decisions such as hiring new sta or re-assignment of the tasks), tactical (weekly planning management and adjustments), or operational level (on-line problems, real-time decisions).

In (Heching et al., 2019), a Benders decomposition is used to solve the re-planning problem whenever there is a change in the pool of patients. In the first step, the assignment problem is solved with a linear program then the tours are built in the second step with constraint programming. The objective is to maximize the number of visits over the week under human continuity constraints. Nickel et al. (2012) also tackle the inclusion of new patients in the system, using an insertion heuristic and an LNS algorithm. They establish an indicator of loyalty between nurses and patients to ensure continuity. Experiments are conducted over real data from a Danish home care provider, with 12 careworkers and 361 patients. Even though continuity of care seems essential in the re-planning process, Gomes & Ramos (2019) consider non-loyalty constraints, inspired by a real case in Portugal, where the careworkers must not be reassigned to the same patients. Travel times are minimized, as is the impact of the perturbations on the initial planning caused by the arrival or departure of patients.

Robust approaches enable decision-makers to forecast perturbations. With cardinality-constrained models, for example, they can keep control over the degree of conservatism in the solutions. In (Cappanera et al., 2014), some patients may cancel their demands, and new patients may enter the system. The cardinality-constrained model limits the number of uncertain demands per tour. Skills requirements, continuity of care, and workload balancing are taken into account under operating cost minimization.

In all these articles, the sta

remains the same.

Some articles also add new decisions to the initial routing and scheduling problem. The possibilities of hiring new careworkers, adding patients to waiting lists, and admitting (or refusing) new patients are considered in (Nasir & Dang, 2018). The main objective is to minimize the costs, notably those related to the

recruitment of new sta
members, the penalties caused by putting patients on the waiting list, etc. In articles tackling the problem at the operational level, several dynamic approaches were implemented. Du et al. (2019) handle emergencies and online cancellations with a memetic algorithm to give a real-time solution that minimizes response time to new demands. Yuan & Jiang (2017) also tackle the real-time problem, but their goal is to minimize the deviation from the initial planning. They minimize the changes in starting times of the services, the changes in the composition of the tours, and then the additional costs implied by late penalties, overtime work, etc. Stability and continuity appear as important factors in the re-planning process, for obvious organizational reasons, but also the convenience of both patients and careworkers.

2.3. Contributions

In this article, we present a method to re-assign and re-route careworkers to face the perturbations caused by the sta
and patient turnover of an HC agency The objective is to re-design a schedule that has become obsolete due to con-secutive perturbations and fixes over the weeks. It is not about finding a quick repair or re-scheduling at every change in the pool of patients and careworkers, but rather about re-optimizing the schedules whenever the successive repairs have become unmanageable and inefficient. Even though the rescheduling is performed on a tactical horizon, we consider that this is a strategic problem since its pivotal constraints (continuity of care) originate from the fact that patients usually stay in HC organizations for a long time, and the goal is to produce a schedule that can be repeated over a long period of time. This angle seems relatively new to tackle the HHCRSP
Continuity of care is essential to ensure the quality of service and patient satisfaction, so we take human continuity into account. As HC is intended for fragile people, patients greatly appreciate consistent service times. To the extent of our knowledge, very little research considers this last feature in HHC literature. We also take several practical constraints into account, such as lunch breaks

and qualifications constraints to stay close to reality, and propose a method that could o er operational and relevant solutions in the field. Unlike most studies, we chose careworker-friendly optimization criteria, and we were able to take advantage of the specific features of the problem to develop an exact method giving solutions in fast computation times for real-life-sized instances.

3. Re-routing and scheduling problem

3.1. Problem statement

Let us consider a set of patients requiring a set of services over a horizon of several days. Each service has a specific duration as well as a fixed starting time. The range of services that can be requested is very broad: it can be medical care, housecleaning, meal preparation, etc. As a consequence, each service is characterized by a level of qualification. Patients may require multiple services per day. To ensure continuity of care, we define a set of known careworkers (from historical schedules of the HC agency) and a tolerance specific to each patient representing the maximum number of di erent careworkers who can visit them during the time horizon. In practice, this tolerance is related directly to the personality or pathology of the patient. The careworkers have di erent levels of skills hierarchically sorted. Overskilled work is allowed: a careworker may be assigned to any service whose required qualification is equal to or smaller than their skill level. Careworkers also have contracts specifying the maximum number of working hours over the consid-ered horizon and di erent availabilities, i.e. time intervals within which they can visit patients.

Whenever HC agencies build a schedule and enforce it for several weeks, numerous variations in the pool of careworkers and/or patients happen and induce changes in this initial schedule. We consider the following perturbations:

- Departure of a patient: many patients leave home care agencies because their condition worsens, they need to go to the hospital, they die, or more luckily, they get better and do not need assistance anymore. Whenever a patient leaves, the careworkers whom they were assigned to might have new gaps in their schedules increasing their idle time, or work much less than their colleagues.
- New patient: whenever new patients enter the structure, they need to be integrated into the existing schedule, in such a way that it does not critically affect the other patients, but also without notably degrading the quality of the schedules for the workers.
- Departure of a careworker: due to difficult working conditions, the turnover is extremely high in the home care field. Whenever careworkers quit, we need to make sure that all of their former patients are re-assigned because the HC agency may not be able to recruit a new careworker right away, let alone with the same characteristics.
- New careworker: the departure of a careworker is frequent, so to avoid being understaffed, home care facilities often bring in new staff members. Due to human continuity constraints, it is sometimes hard to include them in the schedules.

These perturbations most often happen one at a time, and the decision-makers find a quick fix by hand. After several weeks, the perturbations and fixes have accumulated, so the resulting schedule is not only far from the initial one but also far from efficient. At this point, HC agencies need to re-optimize the schedule, while taking into account all these arrivals and departures, but also all the historical data.

3.2. Definitions

The following definitions are extracted from the French Collective Agreements (FCA) (Legifrance, 2012).

- The **amplitude** of a tour is the difference between the starting time of the first service of the day and the ending time of the last service of the day. Note that it does not include the first and last travels of a working day (from the house of the careworker to the first patient's place, and from the last patient's place to the house of the careworker) since they are not considered as working time in the FCA.
- The **idle time** is the time between two consecutive services, travel times excluded.
- The **waiting time** of a tour is the sum of all idle times smaller than 15 minutes. In the FCA, all idle times above 15 minutes are considered as breaks, and thus, not counted as working time.
- The **effective working time** of a tour is the sum of the total time spent alongside the patients during the day, travel times (excluding first and last travel of the day), and waiting times (i.e. idle times smaller than 15 minutes).

3.3. Constraints

We want to fix the initial planning so that all requested services are delivered. We suppose that all the perturbations are known at the time when the re-optimization is done. As we operate at the strategic level, the goal is to build a new schedule that will be used in the long run, whereas an operational strategy would o er a quick fix for a short-term perturbation (punctual absence of a careworker for example). Re-planning for the long term is relevant because it gives careworkers some visibility on their workload, and it also enables HC agencies to spot the need for recruitment or additional training. Even though they cannot control the departure of patients whose health is deteriorating, HC agencies can keep other patients in the system by o ering a good quality of service and a feeling of stability. Moreover, the departure of too many patients creates gaps in the tours. Since those periods not worked are not welcomed by the careworkers, the resulting dissatisfaction may lead to even more departures. The turnover of the staff can be reduced by offering good working conditions.

General constraints

- A tour must start and end at the careworker's home.
- All services must be performed.
- The careworker's skill level must be equal to or superior to the required level of qualifications of the services he or she is assigned to.

Continuity constraints: continuity of care seems to be a key feature for the satisfaction of the patients. As noted above, we consider two types of continuity:

- Temporal continuity: as patients do not like to change their habits, the expected time of the visit (fixed in the initial plan) must not be changed. Consequently, we don't use time windows but fixed starting times.
- Human continuity: patient tolerance τ_p regarding the maximal number of assigned careworkers must not be exceeded. In addition, except for new arrivals, patients already know some careworkers from the previous schedules. Known careworkers should be assigned first. However, since there are variations in the pool of careworkers, such a strict continuity is not always possible. Thus, for each patient p, we calculate their **continuity ratio**: $\frac{\text{number of remaining careworkers known by } p}{\text{tolerance of } p}$. The decision-maker will define a **continuity threshold** such that if a patient's continuity ratio is higher than the continuity threshold, they can only be assigned known careworkers (strict continuity). Otherwise, other careworkers can be assigned, still within the tolerance limits.

Legal constraints: the following constraints are extracted from the French collective agreements (Legifrance, 2012):

- The amplitude of a tour must not exceed the legal value of A_{max}.
- The effective working time of a tour must not exceed the legal value of E_{max}.

- The total effective working time of a careworker k during a week must not exceed his/her contractually defined value h_k.
- A lunch break of at least 45 minutes must be observed during a specific time interval $\mathcal{L}$.
- Each careworker must be granted (at least) a day off during the week.

3.4. Objectives

The turnover of staff can be limited by offering good working conditions. The main criterion of dissatisfaction seems to come from idle times. Indeed, these times, mostly spent in their car between two visits, is extremely badly perceived since it is lost time. The minimization of this lost time can be interpreted as :

1. Minimization of the amplitude of a day: since the services' duration and travel times are incompressible, it is also the minimization of idle times.

2. Minimization of perceived waiting times: the waiting times defined in the FCA are not representative of the time lost during the day because idle times greater than 15 minutes are not counted as waiting times. Moreover, the collective convention considers that when careworkers have a 90-minute break (or longer), they can return home and make the most of this free time. This assumption lies in the fact that, in practice, careworkers are assigned to a limited geographic area around their home. These kinds of breaks, which often happen to be during lunch, are not negatively perceived by the workers, except if there are too many during the day. Thus, the perceived waiting time is computed as the sum of all idle times minus the longest idle time greater than 90 minutes if it exists.

Observing these two criteria will show how the interpretation of lost time can impact the schedules. For comparison purposes and to measure the impact of these two criteria focused on the careworkers, we also study the minimization of travel times, which is the most common objective function in the literature.

4. Solving approach

We decompose this re-optimization problem into two sub-problems. First, we generate all the admissible daily tours that abide by legal and continuity constraints. Then, we select the best subset of tours to build a schedule for all the careworkers over the whole time horizon.
As we work with graphs, the very first phase of our method consists in modeling our problem with a set of graphs such that a path in a graph represents a tour for a careworker. In this section, we detail the algorithm whose global layout is presented in Figure 1. Figure 2 synthesizes the enforcement of the constraints during the different steps. All the notations defined in this section are summed up in Table 1.

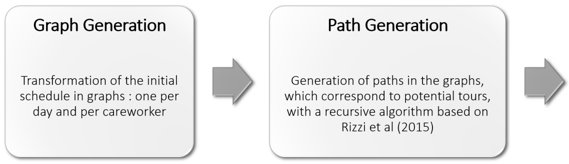

Figure 1: Global layout of the algorithm

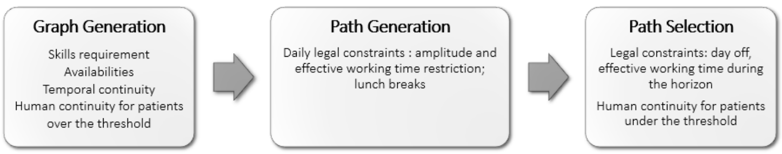

Figure 2: Constraints enforcement

4.1. Example

Let's introduce a simple example to illustrate the solving method throughout the following sections.
Figure 3 illustrates a working day where 11 services are required by di erent

patients. Fixed starting and ending times of services can be read on the horizontal axis. Dotted services (4, 6, 11) require a higher level of qualification than services drawn with a solid line.

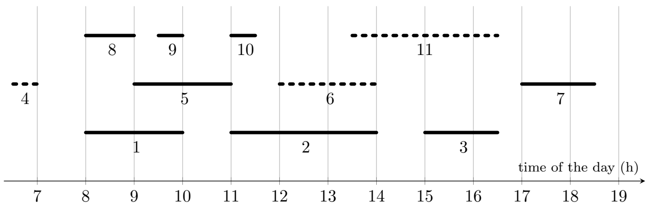

Figure 3: Requested services for a single day

4.2. Graph generation

We generate a set of graphs, one per day and careworker, and we denote by G_{kd} the graph linked to careworker k on day d. Let V_{kd} be the set of vertices containing a source and a sink, and $\mathcal{E}_{kd}$ be the set of arcs of this graph. We build the graphs in such a way that a path from the source to the sink in G_{kd} represents an admissible tour for careworker k on day d.

<u>Vertices</u>: The vertices of G_{kd} are the services that will potentially be part of the tour of careworker k on day d. More precisely, for every service s required by patient p on day d, we add a corresponding vertex v_s in V_{kd} if the following conditions are met:

i) Availability: careworker k is available to perform the service s.
ii) Qualification: careworker k has the required skills for service s.
iii) Human continuity of care: one of the following sub-condition is met:
   - the patient is new in the system or the patient's continuity ratio is smaller than the continuity threshold.
   - careworker k belongs to the set of known careworkers of patient p (condition not valid if the careworker is new).

For every graph G_{kd}, we also add two vertices, a source α_{kd} and a sink β_{kd}, virtually representing the house of the careworker, from which their daily tours must start and end.

Return to the example introduced in Fig. 3. Let's consider a careworker who is available from 8:00 to 17:00, who has the lowest level of qualification and such that human continuity constraints forbid them to perform services 8 and 10. All services are included in the corresponding graph except the following: 7 (availability); 4, 6 and 11(qualification); 8 and 10 (continuity).

Let us now focus on a graph G_{kd} where k and d are chosen arbitrarily. We will momentarily forget the indexes k and d and note the graph G to simplify the notations in this section.

Arcs: We add the following arcs to $\mathcal{E}$:

- αv_i: arcs coming from the source and going to any vertex v_i of the graph
- $v_i\beta$: arcs coming from any vertex v_i of the graph and going to the sink
- v_iv_j: where service i and service j are compatible, meaning that the same careworker can perform both services at the expected times, also considering the travel time between the services.

Example: Let all travel times be 15 minutes. Figure 4 represents the corresponding generated graph for the example introduced in Fig. 3.

Weights:

Let v_i, v_j be two vertices of V_{kd}. Let us assign weights to the arcs v_iv_j based on the definitions of section 3.2:

- $a_{ij} = \delta_i + t_{ij} + w_{ij}$: the amplitude induced by the concatenation of services i and j, where δ_i is the duration of service i and t_{ij} (resp. w_{ij}) is the travel time (resp. idle time) between services i and j.
- $e_{ij} = \delta_i + t_{ij} + w_{ij}^{FCA}$: the effective working time induced by the concatenation of services i and j; where w_{ij}^{FCA} is the waiting time as defined in

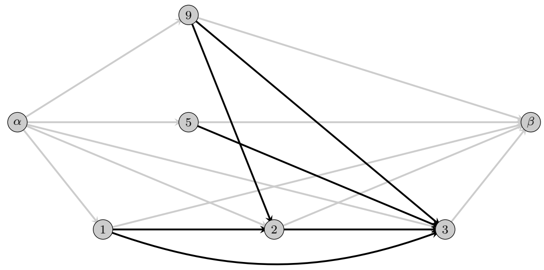

Figure 4: Generated graph for the example in Fig. 3

section 3.2 (= 0 if $w_{ij} \geq 15$ minutes, and w_{ij} otherwise).

Example : Suppose $t_{1.2} = 15min$*. Since* $\delta_1 = 120min$*, we have* $w_{1.2} = 45min$ *;* $w_{1.2}^{FCA} = 0min$ *;* $e_{1.2} = 135min$*.*

4.3. Tour construction

A path from the source to the sink in such a graph represents a sequence of compatible services throughout the day. However, the composition of working days is limited by legal regulations. To be an *admissible tour*, that is to say, a tour that abides by all daily regulations, such a path must have limited weights, in terms of amplitude (A_{max}) and e ective working time (E_{max}). Also, a lunch break must be awarded.

The algorithm `pathGeneration`(G, k, d) (Algorithm 1) lists all admissible paths for a careworker k on a day d. To that end, we compute a lower bound on the e ective working time, the weights of shortest and longest paths on the graph, before calling a recursive algorithm to explore the graph. All these steps are detailed in further subsections.

4.3.1. Lower bounds for the e ective working time: `minBound`

Legal constraints give us upper bounds on the lengths of the paths, but it is also possible to compute lower bounds on the minimal duration (in terms of

Algorithm 1 `pathGeneration`(k, d)

$E_{min} \leftarrow$ MINBOUND$(k, d, \mathcal{S}_d)$ ▷ lower bounds (4.3.1)

for all $v \in V$ **do**:

$x^v_{A_+}$ SHORTAMPLITUDE(v, G) ▷ shortest/longest paths (4.4)

$x^v_{E_+}$ SHORTEFFWT(v, G)

$x^v_{E_-}$ LONGEFFWT(v, G)

return RECPATHGENERATION$(\alpha, \beta, A_{max}, E_{max}, E_{min}, \text{false}, \{\alpha\})$ ▷ see Algorithm 2

effective working time) of a tour. This lower bound, denoted by E_{min}, is specific to every careworker every day and must consequently be computed for every graph.

- We compute the total duration of services required during the day and the maximal theoretical duration of work that can be handled by other workers (based on their availability and their contractual working time). It gives us a lower bound for the service time that must be delivered by the studied careworker.
- When the number of services simultaneously requiring a specific skill matches the number of available careworkers with that skill, we know that every qualified careworker will have to perform one and only one of those services. In consequence, in the set of such overlapping services, the shortest duration gives a lower bound for the service time of the studied careworker. We assume that there are enough skilled workers available to perform such simultaneous services otherwise, the problem would be infeasible.

These di
erent conditions can be combined to find a better lower bound on the e
ective working time of a tour.

Example. Suppose only 3 careworkers are available to perform the services re-

quested in Fig. 3, and only two of them are skilled enough to deliver the dotted services. Services 1, 5, and 9 are overlapping so every careworker will perform exactly one of them. For every careworker, $E_{min} \geq \min\{\delta_1, \delta_5, \delta_9\} = 30min$*. In addition, services 6 and 11 are overlapping. For both skilled careworkers :* $E_{min} \geq \min\{\delta_6, \delta_{11}\} = 2h$*. Since both sets of overlapping services do not intersect,* $E_{min} \geq 2.5h$ *for skilled workers.*
In addition, suppose that both skilled careworkers are contractually limited to 6.5 hours a day. Given that the sum of services duration is 17.5 hours, we can compute for the other careworker that $E_{min} \geq 17.5 - 2 \times 6.5 = 4.5h$*.*

4.4. Shortest/Longest paths: `shortAmplitude`*,* `shortEffWT`*,* `longEffWT`

For every vertex v, we compute in polynomial time (Bellman (1958)):

- the shortest path from v to β in terms of amplitude ($x^v_{A_+}$)
- the shortest path from v to β in terms of effective working time ($x^v_{E_+}$)
- the longest path from v to β in terms of effective working time ($x^v_{E_-}$)

4.5. Construction of admissible paths: `recPathGeneration`

The general algorithm (1) that we designed calls a recursive algorithm (2) inspired by the method given in (Rizzi et al., 2015), which lists all A_+, E_+, E_- length-bounded paths in a graph, where A_+ is an upper bound for the amplitude weight, E_+ is an upper bound for the e ective working time weight, and E_- is a lower bound for the e ective working time weight. For the first call of the function, A_+ and E_+ are set at the legal values defined in the FCA (A_{max} and E_{max}), and E_- is set at E_{min} computed as explained in subsection 4.3.1. Then, they are updated at each recursive call.

We start from a path containing only the source. Then, we check if the addition of a neighbor u can lead to an admissible tour. To that end, we compute the shortest path from u to in terms of amplitude; and we check that the concatenation of this shortest path and the current path ($- u$) has an amplitude shorter than A_{max}. If not, it means that no admissible tour starts by $\alpha - u$ and we can eliminate this sub-path. Similarly, we check the upper and lower bounds on the effective working time. If all bounds are respected, we validate the addition of u to the current path (now $\alpha - u$), and recursively try to add a neighbor of u to the path with the same steps. For that recursive call, we subtract the values induced by the addition of u in the path from the lower and upper bounds.

Finally, the function `isLunchEligible` (constant complexity) returns true if a lunch break can be taken before the start of service u and `IsLunchPossibleAfter` (constant complexity) returns false if no lunch break is possible after service u. This is a simple verification of time constraints regarding the lunch break.

Algorithm 2 `recPathGeneration`$(u, t, A_+, E_+, E_-, b, x_{\alpha u})$

Require:

$x^v_{A_+}$: the shortest path in terms of amplitude from v to β

$x^v_{E_+}$: the shortest path in terms of effective working time from v to β

$x^v_{E_-}$: the longest path in terms of effective working time from v to β

u, t: vertices

A_+: upper bound for the amplitude

E_+: upper bound for the effective working time

E_-: lower bound for the effective working time

b: boolean variable to guarantee lunch break

$x_{\alpha u}$: path from α to u in the graphs

Ensure: all tours x_{ut} (A_+, E_+, E_-)-bounded such that $x_{\alpha u}.x_{ut}$ have a lunch break

if $u = t$ **then**
  return $x_{\alpha u}$

for all $v \in V$ such that $(uv) \in \mathcal{E}$ **do**
  $A'_+ \quad A_+ - a_{uv}$
  $E'_+ \quad E_+ - e_{uv}$
  $E'_- \quad E_- - e_{uv}$
  $b' \quad b \mid\mid$ isLunchEligible(uv)
  if $(A_{x^v_{A_+}} < A'_+)$ & $(E_{x^v_{E_+}} < E'_+)$ & $(E_{x^v_{E_-}} > E'_-)$ & $(b' \mid\mid$ isLunchPossibleAfter$(v))$ **then**
    recPathGeneration$(v, t, A'_+, E'_+, E'_-, b',\ x_{\alpha u}.(uv))$

4.6. Set partitioning

We now have a set $\mathcal{T}_{kd}$ of admissible tours for every careworker every day. We need to pick one tour per day and per careworker such that all the requested services are covered and the constraints are respected. Some constraints were included in the previous steps, such as the daily working time limitations, the skills requirements, or the availabilities. However, we still need to enforce the constraints related to the weekly regulations and some continuity constraints. Let us add to all sets $\mathcal{T}_{kd}$ an empty tour, which represents a day off for the careworker k on day d. This tour will be denoted by the index $j = 0$.

4.6.1. Mathematical formulation of the set partitioning problem

We solve this second problem with a MILP based on a set partitioning model. All the notations are listed in Table 1.

<u>Data and notations</u>

$\mathcal{D}$	$\in \mathbb{N}$	time horizon
$\mathcal{S}_d/S_d$	$\in \mathbb{N}$	set/number of requested services during day d
$\mathcal{T}_{kd}/\ T_{kd}$	$\in \mathbb{N}$	set/number of generated tours for careworker k on day d
h_k	$\in \mathbb{N}$	contractual maximal working time for careworker k over the time horizon
τ_p	$\in \mathbb{N}$	tolerance of patient p
w_{kdj}^{OBJ}	$\in \mathbb{N}$	perceived waiting time of tour j of careworker k on day d
e_{kdj}	$\in \mathbb{N}$	effective working time of tour j of careworker k on day d
a_{kdj}	$\in \mathbb{N}$	amplitude of tour j of careworker k on day d
t_{kdj}	$\in \mathbb{N}$	total travel time of tour j of careworker k on day d
b_{kdj}^s	$\in \{0,1\}$	is equal to 1 if service s is covered by tour j of careworker k on day d, 0 otherwise
y_{dp}^s	$\in \{0,1\}$	is equal to 1 if service s of day d is requested by patient p, 0 otherwise

<u>Decision variables</u>

$$x_{kdj} = \begin{cases} 1 & \text{if careworker } k \text{ performs tour } j \text{ on day } d \\ 0 & \text{otherwise} \end{cases}$$

$$z_{kp} = \begin{cases} 1 & \text{if careworker } k \text{ visits patient } p \text{ during the horizon } \mathcal{D} \\ 0 & \text{otherwise} \end{cases}$$

Objective function

$$\min \sum_{d\in\mathcal{D}} \sum_{k\in\mathcal{K}} \sum_{j\in\mathcal{T}_{kd}} O_{kdj} \times x_{kdj} \tag{1}$$

$$\text{where } O_{kdj} = \begin{cases} w_{kdj}^{OBJ} & \text{to minimize perceived waiting times} \\ a_{kdj} & \text{to minimize amplitudes} \\ t_{kdj} & \text{to minimize travel times} \end{cases}$$

Constraints

$$\sum_{j\in\mathcal{T}_{kd}} x_{kdj} = 1 \quad \forall k \in \mathcal{K}, \quad \forall d \in \mathcal{D} \tag{2}$$

$$\sum_{k\in\mathcal{K}} \sum_{j\in\mathcal{T}_{kd}} b_{kdj}^{s} \times x_{kdj} = 1 \quad \forall d \in \mathcal{D}, \forall s \in \mathcal{S}_d \tag{3}$$

$$\sum_{d\in\mathcal{D}} x_{kd0} \geq 1 \quad \forall k \in \mathcal{K} \tag{4}$$

$$\sum_{d\in\mathcal{D}} \sum_{j\in\mathcal{T}_{kd}} e_{kdj} \times x_{kdj} \leq h_k \quad \forall k \in \mathcal{K} \tag{5}$$

$$z_{kp} \geq y_{dp}^{s} \times \sum_{j\in\mathcal{T}_{kd}} x_{kdj} \times b_{kdj}^{s} \quad \forall k \in \mathcal{K}, \quad \forall d \in \mathcal{D}, \quad \forall s \in \mathcal{S}_d \tag{6}$$

$$\sum_{k\in\mathcal{K}} z_{kp} \leq \tau_p \quad \forall p \in \mathcal{P} \tag{7}$$

$$x_{kdj} \in \{0,1\} \quad \forall d \in \mathcal{D}, \quad \forall k \in \mathcal{K}, \quad \forall j \in \mathcal{T}_{kd} \tag{8}$$

$$z_{kp} \in \{0,1\} \quad \forall k \in \mathcal{K}, \quad \forall p \in \mathcal{P} \tag{9}$$

Our objective is to minimize the perceived waiting times (or the amplitude, or the travel times) for careworkers (1).

We make sure that one and only one tour is assigned to every careworker every day (2), and that all the services requested by the patients are performed during the horizon (3). Constraint (4) ensures that any careworker gets at least one day off during the week. The effective working time of careworker k over the time horizon is limited by constraint (5). The number of different careworkers delivering services to each patient is bounded by its tolerance in the constraints (6) and (7) (see continuity constraint defined in section 3.2). Constraints (8) and (9) state the domains of definition of the variables.

5. Experimentations

We tested our method both on real data sets and on adapted instances from the literature (Bredstöm & Rönnqvist, 2008). The algorithm was implemented in Java, and the MILP was solved with Cplex. All computations were performed on a machine with the following characteristics: Intel ®Core™i7-9850H CPU 2.60GHz and 32 GO of RAM.

5.1. Test instances

5.1.1. Real data: sets 1 to 3

We obtained real data sets from a French Home Care agency We retrieved initial schedules and information about the variations in the pool of patients and careworkers between the time the schedules were built, and several weeks later. The instances' characteristics are summed up in Table 2 (set 1 to 3). We indicate the number of careworkers (K), the number of patients (P), and the number of services (S) that need to be scheduled. Δ_K, Δ_P, and Δ_S represent the variations in the number of careworkers, patients, and services compared to the initial schedule. For example, $\Delta_K = +3/-5$ means that 3 new careworkers entered the agency since the initial schedule was built, and 5 former careworkers left it. For all instances, we work on a weekly horizon : h_k indicates the average contractual maximal working time of careworkers. We also indicate the range of patient tolerance (column τ_p) and the continuity threshold, of which the values were determined with the supervisors of the HC organization.

5.1.2. *Instances from the literature: cases 1 to 6*

We adapted the instances from (Bredström & Rönnqvist, 2008) to test our algorithm. Since we use schedules with fixed service times, we restricted our tests to the instances of the benchmark with fixed time windows. We duplicated the data to obtain weekly demands, and we built a naive schedule to use as the initial schedule. Then we artificially removed some randomly chosen careworkers and brought in new careworkers whose characteristics are similar to those of the pool. We also generated new demands incoming from new patients. We kept the same number of careworkers, patients, and services, to maintain the same workload. The total service duration is much higher than in the real data, so the tours are busier. Preliminary experiments have shown that the instances are unfeasible if we keep the lunch break constraint and the weekly day off. For that reason, both constraints had to be released for the experiments on instances from the benchmark. Still thanks to these preliminary experiments, we adjusted the contractual working times, the continuity threshold, and the tolerances. The reader should also note that because of the way they were created, these instances contain a lot of symmetry that we did not exploit to our advantage in the algorithm since it is generally not the case in practice, at least not on a weekly time horizon. Indeed, the real instances do not present these symmetrical features.

The instances' characteristics are summed up in Table 2 (cases 1 to 6).

5.2. *Computational results*

For each instance, we ran the algorithm on each mono-objective problem. As a reminder, we alternately consider the minimization of amplitude **A**, perceived waiting times **W**, and travel times **T**, expressed in hours in the results. As we have an interest in the impact of the chosen objective on the satisfaction of the worker, for each studied criterion, we also observe the values of the other two criteria. However, since we did not conduct a multi-objective study the presented solutions could be weakly efficient. For each instance, the best values of each objective are highlighted in grey, and we observe the evolution rate of the non-optimized criteria compared to their best value for the same instance (results in percentage). For example, the evolution rate of A is computed as follows:

$$\Delta_A = \frac{\text{best value of A} - A}{\text{best value of A}}$$

The column **Previous plan** shows the values of the amplitude, travel times, and idle times of the initial plan, before the departure and arrival of careworkers and patients. These numbers must be interpreted carefully as the tours are not performed by the same careworkers, and do not contain the same services. Therefore, we cannot directly compare the di erent values, but since the HC organization tries to maintain the same level of workload, we can compare the orders of magnitude. We can observe that the values are closer to the values we obtain when minimizing the amplitude.

Since we developed an exact approach, we can compare the performance of our algorithm with those obtained with a MILP solved with Cplex, using the formulation in (Martinez, 2020) (see Appendix A). The computation times (in seconds) of our algorithm and Cplex on the MILP formulation are expressed in the columns CPU and CPU_{MILP}. The last two colums show the number of variables and constraints of the set partitioning problem of our algorithm.

On the first line of Table 3, we can read the results for set 1 when we minimize the amplitude. It reaches 129.2 hours, and we observe the values of the other criteria: 25.9 hours of perceived waiting time and 11.5 hours of travel time. The second (respectively third) line of the table displays the values of the optimal solution when we minimize the perceived waiting times (resp. travel times).

For all instances, our algorithm manages to reach the optimal solution in less than 5 minutes. The computation times needed to reach the optimal solutions are much faster than the MILP (up to 15 minutes for case 6).

For real instances (sets 1 to 3), we can note that the amplitude and perceived waiting time increase drastically and reach their highest values when the optimized criterion is the total travel time. Travel times are at their maximum when the optimized criterion is the amplitude. On large instances built from the benchmark (cases 5 and 6), we notice the same behavior concerning perceived waiting times. However, travel times and amplitude reach their highest value when the perceived waiting time is minimized.
The variations in travel times are less noticeable in small cases (1 to 4) because the symmetry and the small size of these instances do not leave as many possibilities for re-scheduling.
As a reminder, the amplitude is the sum of service times, travel times, and all idle times. As the total duration of care cannot be shortened, minimizing the amplitude is equivalent to minimizing idle times and travel times. It should be noted that in the calculation of the amplitude, all idle times are taken into account, whereas in the calculation of the perceived waiting time, we deduct the longest gap greater than 90 minutes (if it exists). We can note that in case 4, the amplitude stays the same whatever criterion is minimized, and so does the sum of travel times and perceived waiting times because no gaps greater than 90 minutes were created in the tours.

Overall, we observe that perceived waiting times and travel times are far more impacted by the choice of the objective than the amplitude, even though its variation may be significant, particularly for real instances. The worst variations occur on perceived waiting times when we minimize travel times.
Eventually we also notice that the worst variations on all three criteria occur for real instances. The time range within which the services are requested is broader in real-life cases. Therefore, in those instances, working days may start earlier and end later, so the amplitude is more likely to vary, and so are idle times. Also, services are shorter and workers have lower weekly working times

in real instances. Thus, the tours are less compact and the variability is higher.

We show that the minimization of travel times (one of the most studied criteria in the literature) is predominant for the satisfaction criteria. We modeled the satisfaction through the minimization of amplitude and perceived waiting times. Since the satisfaction of careworkers has a substantial impact on the proper execution of the schedules, minimizing travel times may not be the best option for the employer when the goal is to preserve stability in the staff.

5.3. Discussions

Even though we only study mono-objective problems, we can see that for case 4, the best solution for criteria A is weakly efficient. Using a lexicographic approach would enable us to discard potentially weakly efficient solutions.

Regarding continuity of care, both the temporal and the human aspects are guaranteed in the constraints. Since we modeled it as strict constraints, we do not have any flexibility on the starting time or the patient tolerance. This work could be extended to study a tradeoff between continuity of care, a crucial element of patients' satisfaction, and careworkers' satisfaction.

6. Conclusions and perspectives

In this paper, we tackled the problem of re-assigning careworkers and rescheduling the tours when the turnover of the staff and the patients made plannings infeasible or inecient. Not only we considered legal constraints based on the French collective agreements, but we also took into account the social aspects of Home Care planning with an emphasis on the continuity of care. Two aspects of continuity were preserved: human continuity and temporal continuity, which is rarely studied, to the best of our knowledge.
When most studies present approached solutions or exact solutions to smaller instances, we could design an exact algorithm with fast computation times, able to solve real-life-sized instances.

Our method was designed to optimize different criteria, whether it is a classic criterion such as travel times or more specific objectives related to the satisfaction of careworkers and workplace wellness. The particular features of the real-life case make it easier to get exact solutions. However, with more services, or fewer restrictions on continuity of care, the complete exploration of the graph would be impossible. To solve even larger instances or to schedule over a longer horizon, it could be interesting to work on a heuristic version of the algorithm. It implies the computation of new lower bounds on the paths and a partial exploration of the graph to reduce the number of generated tours. Once the tours are generated, a heuristic can also be used to solve larger set partitioning problems.

References

Bard, J., Shao, Y., & Jarrah, A. (2014). A sequential GRASP for the therapist routing and scheduling problem. *Journal of Scheduling*, *17*, 109–133.

Begur, S., Miller, D., & Weaver, J. (1997). An integrated spatial DSS for scheduling and routing home-health-care nurses. *Interfaces*, *27*, 35–48. doi:10.1287/inte.27.4.35.

Bellman, R. (1958). On a routing problem. *Quarterly of Applied Mathematics*, *16*, 87–90. URL: https://www.ams.org/qam/1958-16-01/S0033-569X-1958-0102435-2/. doi:10.1090/qam/102435.

Borsani, V., Matta, A., Beschi, G., & Sommaruga, F. (2006). A home care scheduling model for human resources. *International Conference on Service Systems and Service Management (ICSSSM), Troyes, France*, *1*, 449–454.

Bredström, D., & Rönnqvist, M. (2008). Combined vehicle routing and scheduling with temporal precedence and synchronization constraints. *European Journal of Operational Research*, *191*, 19–31.

Cappanera, P., Scutellà, M. G., & Visintin, F. (2014). Home Care Services Delivery: Equity Versus Efficiency in Optimization Models. In A. Matta,

J. Li, E. Sahin, E. Lanzarone, & J. Fowler (Eds.), *Proceedings of the International Conference on Health Care Systems Engineering* Springer Proceedings in Mathematics & Statistics (pp. 1–13). Springer International Publishing.

Carello, G., Lanzarone, E., & Mattia, S. (2018). Trade-off between stakeholders' goals in the home care nurse-to-patient assignment problem. *Operations Research for Health Care*, *16*, 29–40. URL: http://www.sciencedirect.com/science/article/pii/S2211692316300716. doi:10.1016/j.orhc.2017.12.002.

Cheng, E., & Rich, J. (1998). A home health care routing and scheduling problem. *Technical report*, . URL: https://scholarship.rice.edu/handle/1911/101899.

Cissé, M., Yalçındağ, S., Kergosien, Y., Şahin, E., Lenté, C., & Matta, A. (2017). OR problems related to Home Health Care: A review of relevant routing and scheduling problems. *Operations Research for Health Care*, *13-14*, 1–22. URL: http://www.sciencedirect.com/science/article/pii/S2211692316300157. doi:10.1016/j.orhc.2017.06.001.

Cordeau, J.-F., & Laporte, G. (2001). A Tabu Search Algorithm For The Site Dependent Vehicle Routing Problem With Time Windows. *INFOR: Information Systems and Operational Research*, *39*, 292–298. URL: https://doi.org/10.1080/03155986.2001.11732443. doi:10.1080/03155986.2001.11732443.

Decerle, J., Grunder, O., Hajjam El Hassani, A., & Barakat, O. (2019). A hybrid memetic-ant colony optimization algorithm for the home health care problem with time window, synchronization and working time balancing. *Swarm and Evolutionary Computation*, *46*, 171–183. doi:10.1016/j.swevo.2019.02.009.

Di Mascolo, M., Martinez, C., & Espinouse, M.-L. (2021). Routing and scheduling in Home Health Care: A Literature Survey and Bibliometric Analysis. *Computers & Industrial Engineering*, (p.

107255). URL: https://www.sciencedirect.com/science/article/pii/S0360835221001595. doi:10.1016/j.cie.2021.107255.

Du, G., Zheng, L., & Ouyang, X. (2019). Real-time scheduling optimization considering the unexpected events in home health care. *Journal of Combinatorial Optimization*, *37*, 196–220. doi:10.1007/s10878-017-0220-3.

Fikar, C., & Hirsch, P. (2015). A matheuristic for routing real-world home service transport systems facilitating walking. *Journal of Cleaner Production*, *105*, 300–310.

Fikar, C., & Hirsch, P. (2017). Home health care routing and scheduling: A review. *Computers & Operations Research*, *77*, 86–95.

Gomes, M. I., & Ramos, T. R. P. (2019). Modelling and (re-)planning periodic home social care services with loyalty and non-loyalty features. *European Journal of Operational Research*, *277*, 284–299. URL: http://www.sciencedirect.com/science/article/pii/S0377221719300967. doi:10.1016/j.ejor.2019.01.061.

Grenouilleau, F., Legrain, A., Lahrichi, N., & Rousseau, L.-M. (2019). A set partitioning heuristic for the home health care routing and scheduling problem. *European Journal of Operational Research*, *275*, 295–303. URL: http://www.sciencedirect.com/science/article/pii/S0377221718309500. doi:10.1016/j.ejor.2018.11.025.

Grieco, L., Utley, M., & Crowe, S. (2020). Operational research applied to decisions in home health care: A systematic literature review. *Journal of the Operational Research Society*, (pp. 1–32). URL: https://orsociety.tandfonline.com/doi/full/10.1080/01605682.2020.1750311. doi:10.1080/01605682.2020.1750311. Publisher: Taylor & Francis.

Heath, S. (2017). Using Home Health Care to Improve the Patient Experience. URL: https://patientengagementhit.com/news/using-home-health-care-to-improve-the-patient-experience.

Heching, A., Hooker, J. N., & Kimura, R. (2019). A Logic-Based Benders Approach to Home Healthcare Delivery. *Transportation Science*, *53*, 510–522. URL: http://pubsonline.informs.org/doi/abs/10.1287/trsc.2018.0830. doi:10.1287/trsc.2018.0830.

Kovacs, A. A., Golden, B. L., Hartl, R. F., & Parragh, S. N. (2014). The Generalized Consistent Vehicle Routing Problem. *Transportation Science*, *49*, 796–816. URL: https://pubsonline.informs.org/doi/abs/10.1287/trsc.2014.0529. doi:10.1287/trsc.2014.0529. Publisher: INFORMS.

Kovacs, A. A., Parragh, S. N., & Hartl, R. F. (2015). The multi-objective generalized consistent vehicle routing problem. *European Journal of Operational Research*, *247*, 441–458. URL: https://www.sciencedirect.com/science/article/pii/S0377221715005445. doi:10.1016/j.ejor.2015.06.030.

Lahrichi, N., Lanzarone, E., & Yalçındağ, S. (2022). A First Route Second Assign decomposition to enforce continuity of care in home health care. *Expert Systems with Applications*, *193*, 116442. URL: https://www.sciencedirect.com/science/article/pii/S0957417421017279. doi:10.1016/j.eswa.2021.116442.

Legifrance (2012). Convention collective nationale des entreprises de services à la personne du 20 septembre 2012 - Texte de base. URL: https://www.legifrance.gouv.fr/affichIDCCArticle.do;jsessionid=1133A88F6CF2A4DE8D36FD10BF728C52.tplgfr42s_1?idArticle=KALIARTI000026943205&cidTexte=KALITEXT000026943196&dateTexte=29990101&categorieLien=id.

Martinez, C. (2020). *Considération de la dimension humaine dans l'optimisation de tournées de soins et services à domicile soumises à des perturbations*. Doctoral dissertation Université Grenoble Alpes. URL: https://tel.archives-ouvertes.fr/tel-03124120.

Mor, A., & Speranza, M. G. (2020). Vehicle routing problems over time: a survey. *4OR*, *18*, 129–149. URL: https://doi.org/10.1007/s10288-020-00433-2. doi:10.1007/s10288-020-00433-2.

Mosquera, F., Smet, P., & Vanden Berghe, G. (2019). Flexible home care scheduling. *Omega*, *83*, 80–95. doi:10.1016/j.omega.2018.02.005.

Nasir, J. A., & Dang, C. (2018). Solving a More Flexible Home Health Care Scheduling and Routing Problem with Joint Patient and Nursing Staff Selection. *Sustainability*, *10*, 0148. doi:10.3390/su10010148.

Nickel, S., Schröder, M., & Steeg, J. (2012). Mid-term and short-term planning support for home health care services. *European Journal of Operational Research*, *219*, 574–587.

Rizzi, R., Sacomoto, G., & Sagot, M.-F. (2015). Efficiently Listing Bounded Length st-Paths. In *Combinatorial Algorithms* Lecture Notes in Computer Science (pp. 318–329). Springer International Publishing.

Rogers, K. (2018). As the US population ages, the need for home health-care workers skyrockets. *CNBC*, . URL: https://www.cnbc.com/2018/05/31/as-the-us-population-ages-the-need-for-home-health-care-workers-skyrockets.html.

Tellez, O., Daguet, L., Lehuédé, F., Monteiro, T., Osorio Montoya, G., Péton, O., & Vercraene, S. (2020). A stakeholder oriented approach to the optimization of transports of people with disabilities. *Supply Chain Forum: An International Journal*, *21*, 93–102. URL: https://doi.org/10.1080/16258312.2020.1768435. doi:10.1080/16258312.2020.1768435. Publisher: Taylor & Francis _eprint: https://doi.org/10.1080/16258312.2020.1768435.

Trautsamwieser, A., & Hirsch, P. (2011). Optimization of daily scheduling for home health care services. *Journal of Applied Operational Research*, *3*, 124–136.

Yuan, B., & Jiang, Z. (2017). Disruption Management for the Real-Time Home Caregiver Scheduling and Routing Problem. *Sustainability*, *9*, 2178. doi:10.3390/su9122178.

Appendix A. MILP for the routing problem

This MILP is from the work of Martinez (2020)
We introduce two dummy services, α and β, to model the start and end of a route. P_σ is the set of patients under the continuity threshold and $\mathcal{S}_p$ is the set of services required by patient p. All the other notations are introduced in the article.

Decision variables

$$x_{ijk} = \begin{cases} 1 & \text{if careworker } k \text{ consecutively performs services } i \text{ and } j \\ 0 & \text{otherwise} \end{cases}$$

$$b_{ijk} = \begin{cases} 1 & \text{if the longest break greater than 90 minutes of careworker } k \\ & \text{is between services } i \text{ and } j \\ 0 & \text{otherwise} \end{cases}$$

$$l_{ijk} = \begin{cases} 1 & \text{if the lunchbreak of careworker } k \text{ is between services } i \text{ and } j \\ 0 & \text{otherwise} \end{cases}$$

$$z_{kp} = \begin{cases} 1 & \text{if careworker } k \text{ performs at least one service to patient } p \\ 0 & \text{otherwise} \end{cases}$$

Objective functions

$$\min \sum_{k \in \mathcal{K}} \sum_{d \in \mathcal{D}} \sum_{i \in S_d} g_i \times x_{i\beta k} - f_i \times x_{\alpha i k} \tag{A.1}$$

$$\min \sum_{k \in K} \sum_{i \in S} \sum_{j \in S} t_{ij} \times x_{ijk} \tag{A.2}$$

$$\min \sum_{k \in K} \sum_{i \in S} \sum_{j \in S} w_{ij} \times (x_{ijk} - b_{ijk}) \tag{A.3}$$

Constraints

$$\sum_{i \in \mathcal{S}_d} x_{\alpha ik} \leq 1 \quad \forall k \in \mathcal{K}, \quad \forall d \in \mathcal{D} \tag{A.4}$$

$$\sum_{i \in \mathcal{S}_d} x_{i\beta k} \leq 1 \quad \forall k \in \mathcal{K}, \quad \forall d \in \mathcal{D} \tag{A.5}$$

$$\sum_{j \in \mathcal{S}^*} x_{ijk} = \sum_{j \in \mathcal{S}^*} x_{jik} \quad \forall k \in \mathcal{K}, \quad \forall i \in \mathcal{S} \tag{A.6}$$

$$\sum_{k \in \mathcal{K}} \sum_{j \in \mathcal{S}^*} x_{ijk} = 1 \quad \forall i \in \mathcal{S} \tag{A.7}$$

$$\sum_{i \in \mathcal{S}^*} x_{ijk} \times (q_k - \rho_j) \geq 0 \quad \forall j \in \mathcal{S}, \quad \forall k \in \mathcal{K} \tag{A.8}$$

$$\sum_{j \in \mathcal{S}^*} x_{ijk} = 0 \quad \forall i \in \mathcal{S} \text{ such that } [f_i, g_i] \not\subset I_k \quad \forall k \in \mathcal{K} \tag{A.9}$$

$$\sum_{i \in \mathcal{S}_p{}^*} \sum_{j \in \mathcal{S}_p} \sum_{k \in \hat{\mathcal{K}}_p} x_{ijk} = 0 \quad \forall p \in \mathcal{P} \tag{A.10}$$

$$x_{ijk} \times (f_j - g_i - t_{ij}) \geq 0 \quad \forall k \in \mathcal{K}, \quad \forall i, j \in \mathcal{S} \tag{A.11}$$

$$\sum_{i \in \mathcal{S}_d} \sum_{j \in \mathcal{S}_d} b_{ijk} \leq 1 \quad \forall k \in \mathcal{K}, \quad \forall d \in \mathcal{D} \tag{A.12}$$

$$b_{ijk} \leq x_{ijk} \times \max(0; 91 - f_j + g_i + t_{ij})) \quad \forall k \in \mathcal{K}, \quad \forall i, j \in \mathcal{S} \tag{A.13}$$

$$\sum_{i \in \mathcal{S}^*} \sum_{j \in \mathcal{S}^*} x_{ijk} \times e_{ij} \leq h_k \quad \forall k \in \mathcal{K} \tag{A.14}$$

$$\sum_{i \in \mathcal{S}_d{}^*} \sum_{j \in \mathcal{S}_d{}^*} x_{ijk} \times e_{ij} \leq E_{max} \quad \forall k \in \mathcal{K}, \quad \forall d \in \mathcal{D} \tag{A.15}$$

$$\sum_{i \in \mathcal{S}_d} (g_i \times x_{i\beta k} - f_i \times x_{\alpha ik}) \leq A_{max} \quad \forall k \in \mathcal{K}, \quad \forall d \in \mathcal{D} \tag{A.16}$$

$$\sum_{i \in \mathcal{S}} x_{\alpha ik} < D \quad \forall k \in \mathcal{K} \tag{A.17}$$

$$\sum_{(i,j) \in {\mathcal{S}_d}^{*2} \cap \mathcal{L}_{ij}} l_{ijk} = 1 \quad \forall k \in \mathcal{K}, \quad \forall d \in \mathcal{D} \tag{A.18}$$

$$l_{ijk} \leq x_{ijk} \quad \forall k \in \mathcal{K}, \quad \forall i, j \in \mathcal{S}^* \tag{A.19}$$

$$z_{kp} \geq \sum_{i \in \mathcal{S}^*} x_{ijk} \quad \forall p \in \mathcal{P} \quad \forall k \in \mathcal{K} \quad \forall j \in \mathcal{S}_p \tag{A.20}$$

$$\sum_{k \in \mathcal{K}} z_{kp} \leq \tau_p \quad \forall p \in \mathcal{P} \tag{A.21}$$

$$\sum_{i \in \mathcal{S}^*} \sum_{k \in \mathcal{K} \setminus \mathcal{K}_p} x_{ijk} = 0 \quad \forall j \in \mathcal{S}_p \quad \forall p \notin \mathcal{P}_\sigma \tag{A.22}$$

$$x_{ijk} \in \{0, 1\} \quad \forall i \in \mathcal{S}, \quad \forall j \in \mathcal{S}, \quad \forall k \in \mathcal{K} \tag{A.23}$$

$$b_{ijk} \in \{0, 1\} \quad \forall i \in \mathcal{S}, \quad \forall j \in \mathcal{S}, \quad \forall k \in \mathcal{K} \tag{A.24}$$

$$l_{ijk} \in \{0, 1\} \quad \forall i \in \mathcal{S}, \quad \forall j \in \mathcal{S}, \quad \forall k \in \mathcal{K} \tag{A.25}$$

$$z_{kp} \in \{0, 1\} \quad \forall k \in \mathcal{K}, \quad \forall p \in \mathcal{P} \tag{A.26}$$

The objective function A.1 minimizes the sum of all amplitudes. The second objective A.2 minimizes travel times, and the third objective (A.3) minimizes idle times (except for the longest break greater than 90 minutes for every careworker) (see equations A.12, A.13).

Routes start and end at careworkers' homes A.4 et A.5. Constraint A.6 guarantees flow conservation.

All services must be performed A.7, and compatibility is checked between consecutive services (A.11). The availability(A.9)and qualification of selected performer is required (A.8), as well as incompatibilities (A.10).

Legal constraints are modeled by equationsA.14 to A.17: limited effective working time during the horizon, limited effective working time per day, limited amplitude of a working day, rest day. Every worker must be granted a lunch break during time interval $\mathcal{L}$ (A.18, A.19). Human continuity is guaranteed by A.20, A.21 et A.22.

Finally, equations A.23 to A.26 state the domains of definition of the variables.

Table 1: Notations of the problem

Notation	Definition
Graph Generation	
$G_{kd}(V_{kd}, \mathcal{E}_{kd})$	graph generated for careworker k and day d
α_{kd}/ β_{kd}	source/sink of G_{kd}
δ_i	duration of service i
t_{ij}	travel time between patients requiring service i and service j
w_{ij}	idle time between service i and service j
w_{ij}^{FCA}	waiting time between service i and service j
e_{ij}	effective working time implied by the successive execution of service i and j
Path generation in G_{kd}	
A_+	upper bound for the amplitude weight
E_+ / E_-	upper/lower bound for the effective working time weight
$x_{A_+}^v$	the shortest path in terms of amplitude from $v \in V_{kd}$ to β_{kd}
$x_{E_+}^v$ / $x_{E_-}^v$	the shortest/longest path in terms of effective working time from $v \in V_{kd}$ to β_{kd}
A_{max}/E_{max}	maximum daily amplitude/effective working time
A_x /E_x	amplitude/effective working time of path x
Path selection - MILP	
$\mathcal{D}$	set of days in the time horizon
$\mathcal{K}$	set of careworkers
$\mathcal{P}$	set of patients
$\mathcal{S}_d$	number/set of services on day d
$\mathcal{T}_{kd}$	set of admissible tours for careworker k on day d
h_k	contractual maximal working time for careworker k over the time horizon
w_{kdj}^{OBJ}	perceived waiting time of tour j of $\mathcal{T}_{kd}$
t_{kdj}	total travel time of tour j of $\mathcal{T}_{kd}$
e_{kdj}	total effective working time of tour j of $\mathcal{T}_{kd}$
a_{kdj}	amplitude of tour j of $\mathcal{T}_{kd}$
b_{kdj}^s	equals 1 if service s is covered by tour j of careworker k on day d, 0 otherwise
y_{dp}^s	equals 1 if service s of day d is requested by patient p, 0 otherwise
x_{kdj}	equals 1 if careworker k performs tour j on day d, 0 otherwise
z_{kp}	equals 1 if careworker k visits patient p during $\mathcal{D}$, 0 otherwise

Table 2: Instances characteristics

Instance	Real data	ΔK	K	ΔP	P	ΔS	S	τ_p	continuity threshold	$\overline{h_k}$
set 1	$\surd$	+2/-3	5	+0/-1	11	+0/-22	85	[3; 6]	0,5	21,1
set 2	$\surd$	+2/-3	8	+2/-7	26	+10/-52	156	[3; 6]	0,5	29,1
set 3	$\surd$	+3/-5	15	+7/-10	92	+33/-16	337	[3; 6]	0,3	26,6
case 1		+1/-1	4	+2/-2	20	+14/-14	140	[2; 4]	0,5	70
case 2		+1/-1	4	+2/-2	20	+14/-14	140	[2; 4]	0,5	70
case 3		+1/-1	4	+2/-2	20	+14/-14	140	[2; 4]	0,5	70
case 4		+1/-1	4	+2/-2	20	+14/-14	140	[2; 4]	0,5	70
case 5		+2/-2	10	+5/-5	50	+35/-35	350	[2; 4]	0,7	90
case 6		+2/-2	10	+5/-5	50	+35/-35	350	[2; 4]	0,7	90

Table 3: Computational results

Instance	Objective	Previous plan	A (h)	W (h)	T (h)	ΔA (%)	ΔW (%)	ΔT (%)	CPU (s)	CPU_{MILP} (s)	#var	#constr
set 1	A	120,4	129,2	25,9	11,5	-	254,8	219,4	3,59	18,3	30429	521
	W	23,2	188,7	7,3	8,5	46,1	-	136,1	3,42			
	T	8,3	215,7	47,1	3,6	67,0	545,2	-	3,53			
set 2	A	290,7	239,7	43,4	18,8	-	329,7	121,2	44,6	122	902068	2428
	W	39,3	366,8	10,1	19,5	53,0	-	129,4	58,9			
	T	13	402,5	65,6	8,5	67,9	549,5	-	45,2			
set 3	A	662,5	490,4	91,9	22,6	-	151,8	133,0	88	1084	865283	10527
	W	90,4	747,5	36,5	21	52,4	-	116,5	100			
	T	20,4	793,4	152	9,7	61,8	316,4	-	63			
case 1	A	-	251,6	21,6	37,5	-	0,6	15,0	2,9	34,1	45420	752
	W	-	251,6	21,5	37,6	0,0	-	15,3	2,6			
	T	-	260,3	29,9	32,6	3,5	39,3	-	2,3			
case 2	A	-	264,8	14,9	43	-	88,6	27,2	8,9	44,3	121892	752
	W	-	279,8	7,9	42,5	5,7	-	25,7	6,5			
	T	-	273,8	32,3	33,8	3,4	308,9	-	5,8			
case 3	A	-	238,0	7,1	37,3	-	20,3	5,4	1,4	19,4	27906	752
	W	-	268,0	5,9	38,7	12,6	-	9,3	1,4			
	T	-	265,9	6,6	35,4	11,7	11,9	-	1,4			
case 4	A	-	216,4	11,5	35,8	-	57,5	7,2	1,5	24	28425	752
	W	-	216,4	7,3	40	0,0	-	19,8	1,7			
	T	-	216,4	13,9	33,4	0,0	90,4	-	1,8			
case 5	A	-	567,9	41,5	100,7	-	42,1	21,2	162	798	589267	3980
	W	-	578,8	29,2	112,8	1,9	-	35,7	61,5			
	T	-	574,0	65,8	83,1	1,1	125,3		41,8			
case 6	A	-	536,8	20,6	111,4	-	50,4	21,5	252	873	1678735	3980
	W	-	626,5	13,7	115,7	16,7	-	26,2	218			
	T	-	612,7	76,6	91,7	14,1	459,1	-	133			

Highlights

- Continuity of care is a key element in rescheduling problems for HC organizations
- Minimizing travel times generally leads to unacceptable tours for careworkers
- Real-life features are used to efficiently model HC scheduling problems with graphs

Credit author statement:

Cléa Martinez: Conceptualization, Investigation, Writing- Original Draft

Marie-Laure Espinouse: Conceptualization, Supervision, Writing,-Reviewing and Editing

Maria Di Mascolo: Conceptualization, Supervision, Writing-Reviewing and Editing