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A comparative viewpoint in the control of switched power converters

C. Valentin, H. Jerbi and F. Omri

Abstract—In this paper, three control strategies of switched power converters are confronted: a discrete stabilizing control strategy based on a common quadratic Lyapunov function (DSC-CLF)[1], a discrete time optimal control strategy (DOC-T) [2] and a continuous stabilizing control law based on interconnection and damping assignment (CSC-IDA) [14]. In the three first sections, the three methods are briefly recalled. The reader can refer to [1], [2] or [14] for a more detailed description. The contribution of this paper lies in the understanding and analysis of the advantages and drawbacks of each method to control an induction heating appliance.

I. INTRODUCTION

Switched power converters belong to the class of switched linear systems. A switched (linear) system [9], [18] is made of a family of continuous (linear) subsystems described by some (linear) differential equations and a piecewise constant control law orchestrating the switching between them (it gives the active subsystem at each instant t). Each active subsystem represents one discrete state (mode) of the global switched system. If the state space is \mathbb{R}^2 , the switched system is planar. Research on planar linear switched systems has attracted more and more attention, even if they are the simplest among switched systems. But their simplicity in the mathematical form does not decrease their interest because of their presence in the engineering and industry. One of the challenges on these systems, is their stabilization (see for example [4], [7], [8], [11], [13] and [19]). For more details about switched systems stability and stabilization, see [9].

The search for Lyapunov functions, i.e. functions that can be assigned to represent the energy of the system, is a very important step in stability analysis and stabilization of switched systems. For stabilization, the search for stabilizing control laws can be reduced to a Lyapunov functions search (see [5], [9], [6], [7], [11], [13], [19]). The control objectives (optimality, robustness), sometimes including the physical system constraints, impose how to use these functions in the control (see [12], [16]). Therefore, the search for Lyapunov functions is part of the synthesis of a stabilizing control law. Methods 1 and 2 deal with this idea in different ways. The method 2 is based on geometric properties of these systems in the plane and avoids this research. In the three first sections, the three methods are briefly recalled. In section 5, the three control strategies are confronted on an induction heating appliance.

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II. A DISCRETE STABILIZING CONTROL STRATEGY BASED ON A COMMON QUADRATIC LYAPUNOV FUNCTION (DSC-CLF)

In [1] generic methods are presented to design Lyapunov functions for switched systems using mathematical tools from the plane geometry, for planar linear switched systems with two discrete states (two subsystems) defined as:

$$\dot{x} = A_i x + B_i \quad \text{with } x \in \mathbb{R}^2 \quad \text{and } i \in \{1, 2\} \quad (1)$$

This method aims to stabilize the planar switched system around (Lyapunov stability) or at (asymptotic stability) a desired operating point, x_d , with a switching law defined by an efficient choice of the active subsystem according to specific regions, Ω_i , of the real valued state space: $\Omega_i = \{x \in \mathbb{R}^2 / \text{the subsystem } i \text{ can be active}\}$. The stabilization is achieved in two steps:

- 1) Find a bounded and not empty region of the real valued state space that contains x_d and represents a minimum of energy, V_{min} , such that, once reached by the switched system, a control law such that the continuous state remains in this region exists. This region is called "stability region" of the switched system. Note that, if the minimum of energy is equal to zero, the stability region is reduced to a single point and the system is asymptotically stable.
- 2) Guarantee, outside the stability region, the existence of at least one switching sequence between the two subsystems that lead the switched system to the stability region (or point). The control design can be performed in the following way. Let us consider $q = q_1 - aq_2$, a quadratic function. The switching control of the switched system depends on the sign of q . It is a piecewise continuous signal.

To solve the stabilization problem, a measure of the switched system energy is necessary. The first intuition is to find a candidate-Lyapunov-function (locally positive-definite function decreasing into Ω_i), common to both subsystems and that has a minimum reachable by switching. Since the subsystems are linear, the simplest way is to design a quadratic Lyapunov function. The design of this function is based on the study of the geometric shape of the isotropic surfaces S_i and of the sets Ω_i . It depends on the signature of the quadratic form $y^T(PA_i)y$.

Looking for this function V , is equivalent to look for a matrix P which ensures the switched system stabilization. Note that, if such a function exists, the control strategy that allows switching only on isotropic surfaces, stabilizes the

planar linear switched system, but is not necessarily optimal (with regard to time, energy, ...).

In [1], the case of switched systems made of two stable subsystems is studied and solved. The reader can refer to [1] for more details about this method. The application of the induction heating appliance belongs to this case to the extent that the system is controllable if only two of the three possible configurations are used.

III. A DISCRETE TIME OPTIMAL CONTROL STRATEGY (DOC-T)

The algorithm for the synthesis of a stabilizing control for the switched power converter has six steps [2]:

- 1) Write the switched power converter model in the form: $\dot{x} = f_i(x)$ with $i \in \{1, 2\}$.
- 2) Determine the set E given in definition (3.1) in [2]:
 - If $E \neq \emptyset$, then at least a hybrid limit cycle exists and the SDS can be stabilized around all points of E . Go to step 3.
 - If $E = \emptyset$, no switching sequence exists to stabilize the SDS around any point of the continuous state space. The switched power converter cannot be stabilized.
- 3) After choosing the desired operating point, x_d , determine x_{dE} , the point of E around which a hybrid limit cycle $CC(x_{c1}, x_{c2})$ can be designed:
 - If $x_d \in E$, then $x_{dE} = x_d$.
 - Otherwise, $x_{dE} \in E$ is such that $d(x_{dE}, x_d) = \min_{z \in E} d(z, x_d)$.
- 4) Determine a hybrid limit cycle around x_{dE} that meets the desired specifications. By definition, a hybrid limit cycle whose interior includes x_{dE} always exists but not necessarily one that meets the technological constraints ($\delta_{c1} > \delta_{min}$ and $\delta_{c2} > \delta_{min}$, δ_{min} is the minimum duration time between two successive switchings (dwell time), the state variables are bounded by specific values, the cycle duration time, $\delta_{c1} + \delta_{c2}$, is bounded, etc.). This hybrid limit cycle is not unique.
 - If there is such a hybrid limit cycle, go to Step 5.
 - Otherwise, remove a constraint or choose another point x_{dE} .
- 5) Determine backward (according to definition(4.1) in [2]), all regions from which the hybrid limit cycle can be reached whilst respecting the global operating constraints. If one is interested only in finding a hybrid switching sequence that drives the system from the initial state x_0 to the final hybrid limit cycle, the algorithm is completed as soon as the initial state x_0 is included into one of these regions. If possible, all the regions included in Ω can be calculated and all the possible switching sequences that drive the system from the initial hybrid state to the desired hybrid limit cycle are made known.
- 6) Calculate the switching instants of a reachability sequence found in the previous step to obtain the

shortest total reaching time, t_c .

IV. A CONTINUOUS STABILIZING CONTROL LAW BASED ON INTERCONNECTION AND DAMPING ASSIGNMENT (CSC-IDA)

If we look at a dynamical system as an energy transformation device, a closed loop system maybe decomposed in a controller interconnected with the physical system. Then, the control problem consists in finding a control such that the overall energy function, the interconnections and dissipations inside the closed loop system take a desired form. This approach developed in [14] belongs to passivity based control (PBC) approaches. As soon as electrical circuits are concern, the energy state variables are the charge in the capacitors, q_i , and the flux through the inductances ϕ_j . The port power variables are the input voltage or current. The total stored energy function, H , is: $H = \sum \frac{q_i^2}{2C_i} + \sum \frac{\phi_j^2}{2L_j}$. To make explicit the interconnection and the damping terms, the model of the system is formulated through a Port-Hamiltonian representation. Let us give an example in dimension 2:

$$\begin{bmatrix} \dot{\phi} \\ \dot{q} \end{bmatrix} = [J - R] \frac{\partial H}{\partial x} + g(x)\rho + U$$

with J the internal interconnection structure, R , the damping structure, ρ the input port control signal of the switches, U the input port signal and g the external interconnection structure.

The shaped Hamiltonian formulation of the closed loop system is:

$$\dot{x} = [J_s - R_s] \frac{\partial H_s}{\partial x}$$

with $H_s(0) = 0$.

The control signal ρ is calculated in order that the closed loop system behaves as the shaped Hamiltonian formulation.

V. THREE CONTROL STRATEGIES OF AN INDUCTION HEATING APPLIANCE

An induction heating appliance (induction hob) is made of adaptable-diameter inductors. Currently, one resonant inverter is dedicated to supplying each winding [15]. The current variations in the inductor produce heat energy in the metal vessel placed on the winding. The inductor and the load (vessel) are each equivalent to a resistor and an inductor in series. In the system studied here, the global inductor (R , L) is in series with a capacitor C to compose the resonant inverter (voltage inverter). The voltage source, E , provides adjustable DC voltage through a thyristors, IGBT (Insulated Gate Bipolar Transistor) or diodes rectifier.

An adequat DC voltage is applied to the series RLC circuit, with the control of the opening and closure of the four switches $T1$, $\bar{T}1$, $T2$ and $\bar{T}2$. Three configurations are possible:

- 1) $T1$ closed and $T2$ open,
- 2) $T2$ closed and $T1$ open,

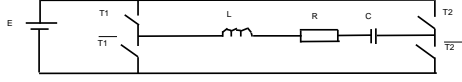


Fig. 1. Diagram of the induction heating appliance

3) $T1$ and $T2$ closed.

The objective is to control the u_C voltage to a desired hybrid limit cycle [2] and stabilize it on it, whatever the load. Let us denote the average value of this limit cycle x_d such that: $x_d = \begin{bmatrix} i_L \\ u_C \end{bmatrix} = \begin{bmatrix} 0 \\ u_{Cd} \end{bmatrix}$

The standard state-space representation of the induction heating appliance, with power state variables, is the following equation (2):

$$\begin{bmatrix} \dot{i}_L \\ \dot{u}_C \end{bmatrix} = \begin{bmatrix} \frac{-R}{L} & \frac{-1}{L} \\ \frac{1}{C} & 0 \end{bmatrix} \begin{bmatrix} i_L \\ u_C \end{bmatrix} + \rho \begin{bmatrix} \frac{2E}{L} \\ 0 \end{bmatrix} - \begin{bmatrix} \frac{E}{L} \\ 0 \end{bmatrix} \quad (2)$$

with ρ the control signal of the switches. If $\rho = 1$, the configuration 1 is active, if $\rho = 0$, the configuration 2 is active and, if $\rho = 0.5$, the configuration 3 is active. The dynamics of these three configurations are given figure 2.

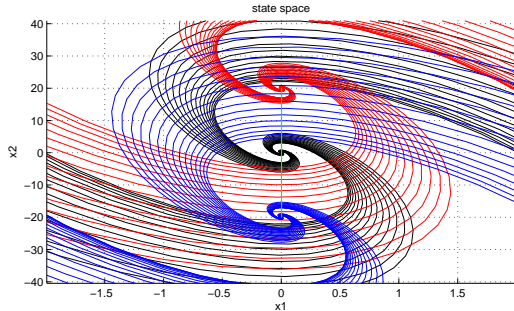


Fig. 2. State-space trajectories of the three configurations of the induction heating appliance (config 1 in red, config 2 in blue and config 3 in black). The closed set of equilibrium points of the convexified model is in green.

The set of all points around which a hybrid limit cycle can be established (named E in \mathbb{R}^2 and \mathcal{E} in \mathbb{R}^n with $n > 2$ in [2]) is the open straight line segment connecting the two equilibrium points of configurations 1 and 2 except the origine (equilibrium point of the configuration 3). The hybrid limit cycle has to respect the technological constraints (minimum duration between two successive switchings, boundedness of the real valued state variables). This set is also the set of equilibrium points of the convexified model made of equation (2) with $\rho \in [0, 1]$ i.e. $\{x \in \mathbb{R}^2 / \begin{bmatrix} 0 \\ -E \end{bmatrix} \leq x \leq \begin{bmatrix} 0 \\ E \end{bmatrix}\}$.

Port-Hamiltonian representation:

$$\begin{bmatrix} \dot{\Phi} \\ \dot{q} \end{bmatrix} = \left[\begin{bmatrix} 0 & -1 \\ 1 & 0 \end{bmatrix} - \begin{bmatrix} R & 0 \\ 0 & 0 \end{bmatrix} \right] \frac{\partial H}{\partial x} + \rho \begin{bmatrix} 2E \\ 0 \end{bmatrix} - \begin{bmatrix} E \\ 0 \end{bmatrix}$$

i.e.

$$\begin{bmatrix} \dot{\Phi} \\ \dot{q} \end{bmatrix} = [J - R] \frac{\partial H}{\partial x} + \rho \begin{bmatrix} 2E \\ 0 \end{bmatrix} - \begin{bmatrix} E \\ 0 \end{bmatrix}$$

$$\text{with } H = \frac{\phi^2}{2L} + \frac{q^2}{2C} \text{ et } x = \begin{bmatrix} \phi \\ q \end{bmatrix}$$

$$x_d = \begin{bmatrix} 0 \\ u_{Cd}C \end{bmatrix}$$

$\rho = 0.5$ in phase 1, $\rho = 1$ in phase 2 and $\rho = 0$ in phase 3. Note that some of the three phases may not appear in the limit cycle.

A hybrid limit cycle around $x_d = [0 \ 10V^T$ with phases 2 et 3 is given figure 3. It respects the technological constraints which impose a minimum duration between two successive switchings. The hybrid limit cycle duration is 0.5 ms, the blue trajectory is 0.13 ms and the red trajectory is 0.37 ms. The maximum switching frequency is 7.7 kHz.

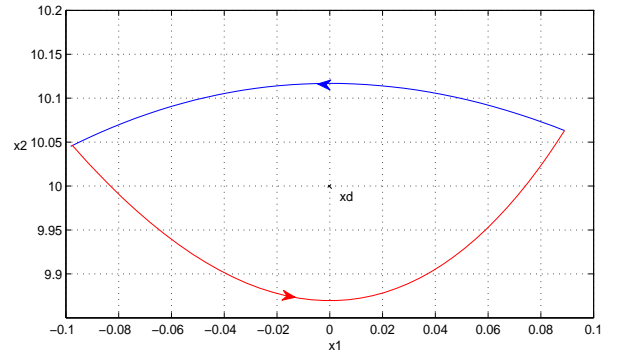


Fig. 3. Hybrid limit cycle for the control of the induction heating appliance around x_d (config 1 in red, config 2 in blue)

With the discrete stabilizing control strategy based on a common quadratic Lyapunov function (DSC-CLF)[1], the goal is then to reach this hybrid limit cycle with the least amount of energy dissipation as possible and to stabilize on it. To design the stability and control around 0 instead of x_d , a change of variable is performed: $y = \begin{bmatrix} \phi \\ q - u_{Cd}C \end{bmatrix}$. Then, a LMI (Linear matrix inequality) gives the matrix P of the quadratic Lyapunov function. As all the configurations are stable, the isotropic surfaces are ellipses and the trajectories are given in figure 4.

With the discrete time optimal control strategy (DOC-T) [2], the goal is then to reach this hybrid limit cycle the fastest as possible. A time optimal control problem is solved. The trajectories are given in figures 5, 7 and 6 with the two other approaches.

To deal with the Interconnection and Damping Assignment Passivity Based Control (IDA-PBC), the convexified model

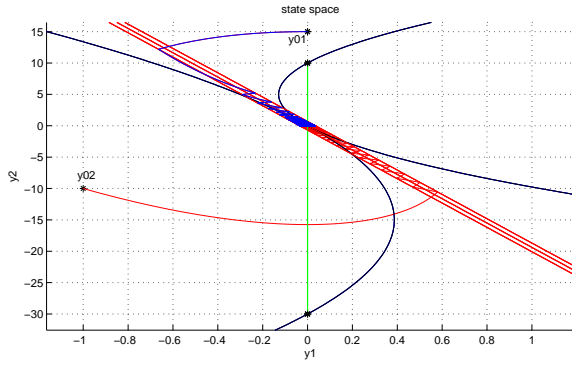


Fig. 4. Isotropic surfaces (black ellipses parts), sliding surfaces with hysteresis (red lines), set of possible equilibrium points (green) and control trajectories to reach a hybrid limit cycle around 0 for the induction heating appliance (from y_{01} in red, from y_{02} in blue)

is used:

$$\dot{x} = \left[\begin{array}{c} \left[\begin{array}{cc} 0 & -1 \\ 1 & 0 \end{array} \right] - \left[\begin{array}{cc} R & 0 \\ 0 & 0 \end{array} \right] \frac{\partial H}{\partial x} + \rho \left[\begin{array}{c} 2E \\ 0 \end{array} \right] - \left[\begin{array}{c} E \\ 0 \end{array} \right] \end{array} \right]$$

with $\rho \in [0, 1]$. The set of points around which it is possible to have a hybrid limit cycle is the set of equilibrium points of the convexified model: $\{x \in \mathbb{R}^2 / i_{Le} = 0 \text{ and } -E < u_{Ce} < E\} = \{x \in \mathbb{R}^2 / \phi = 0 \text{ and } -EC < q < EC\}$.

The shaped Hamiltonian function for PBC is the total stored energy which vanishes at the desired operating point x_d . $H_s = \frac{(q - u_{Cd}C)^2}{2C} + \frac{\phi^2}{2L}$

After the change of variables, we obtain: $H_s = \frac{y_1^2}{2L} + \frac{y_2^2}{2C}$ and

$$\begin{bmatrix} \dot{y}_1 \\ \dot{y}_2 \end{bmatrix} = \begin{bmatrix} -R-1 \\ 1 \ 0 \end{bmatrix} \frac{\partial H_s}{\partial y} + \rho \begin{bmatrix} 2E \\ 0 \end{bmatrix} - \begin{bmatrix} E + u_{Cd} \\ 0 \end{bmatrix}$$

The Interconnection and damping of the shaped Hamiltonian representation are assigned at: $J_s = J$ and $R_s = \begin{bmatrix} \alpha R & 0 \\ 0 & 0 \end{bmatrix}$ with α a tuning parameter to shape the transient behavior.

It follows the static state feedback:

$$\rho = \frac{1}{2E} \left[\frac{R}{L} (1 - \alpha) y_1 \right] + u_{Cd} + E \text{ with } \rho \in [0, 1].$$

The comparative trajectories designed with the three control strategies are given in figures 5, 7 and 6.

	$y_0 = [0 \ 15C]$	$y_0 = [-L \ -10C]$
DOC-T	1.5 ms (magenta)	3.6 ms (yellow)
DSC-CLF	5.6 ms (blue)	6.26 ms (red)
CSC-IDA	4.8 ms (cyan)	5.8 ms (green)

Total time to reach the hybrid limit cycle

The discrete time optimal control strategy (DOC-T) [2] gives the fastest trajectories to reach the hybrid limit cycle. The discrete stabilizing control strategy based on a common quadratic Lyapunov function (DSC-CLF)[1] gives the trajectories with the least amount of energy dissipation as possible but they are the slowest. The Interconnection and Damping Assignment Passivity Based Control (IDA-PBC) gives the trajectories with the most damping. A compromise

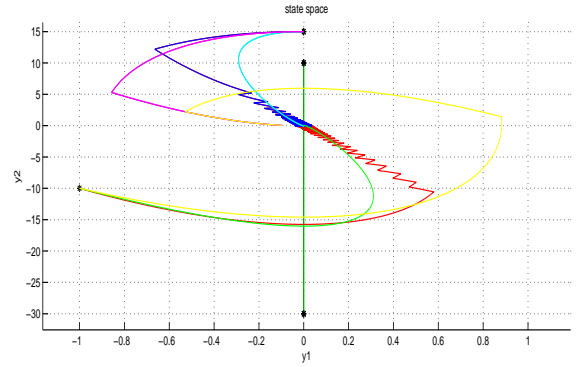


Fig. 5. Comparative state-space trajectories of the three strategies to control the induction heating appliance (respective curves colors are given in the table below)

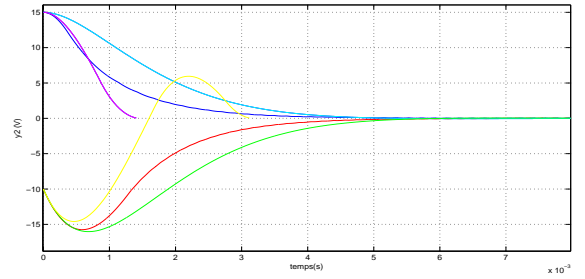


Fig. 6. Comparative capacitor voltage trajectories of the three strategies to control the induction heating appliance (respective curves colors are given in the table below)

between speed and damping may be realized with the tuning parameter α .

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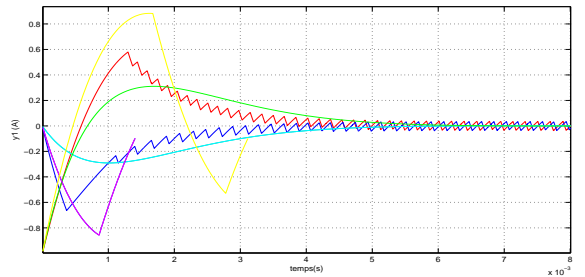


Fig. 7. Comparative inductor current trajectories of the three strategies to control the induction heating appliance (respective curves colors are given in the table below)

VII. CONCLUSIONS AND PERSPECTIVES

In this paper, three control strategies of switched power converters are briefly recalled and confronted: a discrete stabilizing control strategy based on a common quadratic Lyapunov function (DSC-CLF)[1], a discrete time optimal control strategy (DOC-T) [2] and a continuous stabilizing control law based on interconnection and damping assignment (CSC-IDA) [14]. They are applied to an induction heating appliance.

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