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Handling Uncertainties with and Within Digital Twins

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Abstract. The Digital Twin (DT) is often used in environments characterized by uncertainty and complexity, where operating conditions are prone to variability based on external and internal factors. Thus, the literature about DT emphasizes the importance, limitations, and absence of uncertainty quantification. However, there is no explicit review discussing uncertainty in complex systems and within the digital twin model. Such an explicit review could improve the conception, construction, and utilization of DT in environments that are both dynamic and stochastic. Thus, this article aims to (1) describe how a DT can help manage uncertainties in a dynamic system, and (2) explain how DT should deal with uncertainties inside the model.

1 Introduction

Multiple fields paid close attention to digital twins (DTs) during the last decade. The Digital Twin has been extensively considered in the field of manufacturing [1] such as autonomous manufacturing [2] and additive manufacturing [3], and has even been extended to cyber-physical systems [4]. It has also been explored for asset management activities like damage detection and condition monitoring [5] as well as uncertainty quantification [6], and risk assessment [7], and also in the healthcare field [8] for patient pathway tracking in hospitals [9] and even inside operating room [10].

Such systems are dynamic, highly complex, and exhibit stochastic characteristics [11]. They evolve with time, are hard to model, and can deviate from their initial configuration because of random events. Thus, their DT is intended to capture the real system dynamics and adapt to changes that might occur in an uncertain environment. Regardless of how accurate a DT is or how much data is gathered from the monitored system, a DT will always be an imperfect representation. Indeed, a DT systematically suffers from modelling errors and parameter uncertainty. Even when these parameters are known and correct, there is always a mismatch between the real behaviour of the system and the simulated behaviour of the DT because it is only an approximation of the real system.

For all these reasons, the literature abounds with work related to the notion of DT, their associated enabling technologies, and their problems.

The DT is often used in environments characterized by uncertainty and complexity, where working circumstances might change based on external and internal variables [9]. As a result, Z. Liu et al. [13] suggest the definition of ‘a living model that continually adapts to change in the environment or operation using real-time sensory data and can forecast the future of the corresponding physical assets’, whereas Zhuang et al. [14] present the concept as ‘a dynamic model in the virtual world that is fully consistent with its corresponding physical entity in the real world and can simulate its physical counterpart’s characteristics, behaviour, life, and performance in a timely fashion’.

The literature about DT emphasizes the importance, limitations, and absence of uncertainty quantification [15–17]. However, to the best of the authors’ knowledge, there is no clear review regarding uncertainties in the digital and the physical worlds. Such an explicit review could improve the conception, construction, and utilisation of DT in environments that are both dynamic and stochastic. Thus, this article aims to (1) describe how a DT can help manage uncertainties in a dynamic system, and (2) explain how one should deal with uncertainties inside the DT model.

According to this motivation, we investigate the literature to answer the following research questions:

- **RQ1:** What are the types of uncertainties identified in the literature?
- **RQ2:** What is the difference between uncertainties in the physical world and the DT model?
- **RQ3:** What are the used methods to deal with the uncertainties with and within the DT?

The remainder of this paper is structured as follows. In Sect. 2, we present several uncertainty categories. Then, in Sect. 3, we investigate where these uncertainties appear in both the physical system and DT model. In Sect. 4, we describe the methods used to manage these uncertainties with and within the DT. Finally, Sect. 5 is dedicated to the Discussion and Sect. 6 proposes concluding remarks.

2 Types of Uncertainty

The scientific community provides field-specific typologies for uncertainties [18]. Milliken [15] characterizes *uncertainty* in organization theory with state uncertainty as an inability to “understand how components of the environment might be changing”, *effect uncertainty* as “an inability to predict what the nature of the impact of a future state of the environment or environmental change will be on the organization”, and *response uncertainty* as “an inability to predict the likely consequences of a response choice”. In [19], process design and operations uncertainties are divided into model-inherent uncertainty, process-inherent uncertainty, external uncertainty, and discrete uncertainty. The two first are epistemic uncertainties, and the two last are aleatory uncertainties. The article [18] defines uncertainty as “any departure from the unachievable ideal of complete determinist”. They distinguish three dimensions of uncertainty in decision support systems: the

uncertainty's location (the place where the uncertainty occurs), the uncertainty's level (the amount of knowledge available), and the uncertainty's nature (whether the uncertainty is epistemic or aleatory). Finally in the operating room, [20] makes a difference between natural variability and artificial variability (which we would both call aleatory uncertainty in this article). Natural variability is the result of clinical variability, patient flow variability, and professional variability. It cannot be avoided as it is inherent to the healthcare environment. Artificial variability is human-induced (e.g., patient and staff preferences, planning, and scheduling). It is non-random and yet non-predictable.

Other definitions are more global. The paper [21] states that uncertainty is “an individual's perceived inability to predict something accurately” and that it “refers to the psychological state of doubt about what current events mean or what future events are likely to occur”. [22] describes it as “a cognitive state of the individual, resulting from the evaluation of the number of alternatives available to predict future behaviour or alternatives available to explain a past behaviour”. In the field of risk management, the ISO 31000:2009 standard on risk management vocabulary [23] describes uncertainty as “the state, even partial, of deficiency of information related to, understanding or knowledge of, an event, its consequence, or likelihood.” These references agree with the idea that uncertainty is a perceptive phenomenon in which the access to information is crucial [24]. In various scientific fields (e.g., computational engineering [25], decision support [18], and electromechanics [26]), we can distinguish two main types of uncertainty: aleatory uncertainty and epistemic uncertainty.

Aleatory uncertainty is inherent and specific to any physical phenomenon which displays a random behaviour. It is the impact of the input natural variation on the outputs. It is irreducible without changing the studied system itself and is commonly treated with probability theory. Different classifications can be found in the literature depending on the area of application; in production planning, aleatory uncertainty is divided into demand forecast uncertainty, external supply process uncertainty, and internal supply process uncertainty [27]. In the supply chain domain, it includes supply uncertainty, process uncertainty, and demand uncertainty [28]. In the operating room environment [29] identifies four main uncertainties: patient arrival, surgery duration, care requirement, and resource uncertainty.

Epistemic uncertainty stems from imperfect knowledge of the system studied. It can thus be reduced by retrieving more information about the system. Different epistemic uncertainty categorization exists; for instance modelling errors and parameters uncertainties [25], or model uncertainty and data uncertainty [26]. More specifically, uncertainties can be designated as structured if they just affects the values of the model's parameters and not its structure, and as non-structured if it impacts the model's structure [30]. Kennedy and O'Hagan [31] state that epistemic uncertainties stem from two sources: parameters and models.

- Uncertainty due to the parameters refer to:
 - *Parameter uncertainties*: models invariably contain parameters that are quantifiable but are not completely understood or accessible in most circumstances.
 - *Parametric variability*: because inputs cannot be entirely controlled or described, the circumstance in which the model is used may change. A model, on the other

hand, may necessitate the definition of a single deterministic value, which should be adjusted depending on process information.

- Uncertainty due to the model are:
 - *Model discrepancy*: it is understood that even when the parameters are deterministic and ‘really’ known, there will still be mismatches between the model output and the ‘true’ physical process.
 - *Residual variability*: given the same set of inputs, the process may yield various results due to a chaotic or stochastic character. This is frequently caused by insufficiently specified inputs, the inherent random nature of the process, measurement noise, or caused by a lack of understanding or knowledge of the observed system.

[32] suggests another epistemic uncertainty typology and states that it can be divided into model uncertainty and data uncertainty. Model uncertainty includes parameter uncertainty, solution approximation errors, and model form uncertainty. Data uncertainty includes measurement uncertainty and sparse or imprecise. Sometimes aleatory uncertainty is referred to as “variability” and epistemic uncertainty as “uncertainty”.

To illustrate the difference between epistemic and aleatory uncertainty, we suggest a simple example. When throwing a fair coin, we know that the results can be head (50% probability) or tail (50% probability). However, we cannot predict for sure what will be the output of a specific throw. We call aleatory uncertainty the variability output due to the inherent randomness of the studied system. In case the coin is not fair, there will also be epistemic uncertainty as we will not have perfect knowledge of the head/tail frequency. Thus, we will have to model the probability distribution that predicts the output. As a side note, the authors of [33] consider aleatory uncertainty as not uncertainty but as pure variability, and epistemic uncertainty as general uncertainty. This emphasizes the idea that defining the terms used is extremely important for a good comprehension of the problem.

Differentiating these two types of uncertainty is not always necessary. On one hand, in the context of probability theory and mathematical statistics of uncertainty quantification, the same tools are used for: (1) the stochastic modeling of uncertainties, (2) the analysis of uncertainty propagation in a computational model, and (3) the solving of inference or estimation. Thus, it is not necessary to deal with each type of uncertainty separately [26]. On the other hand, aleatory uncertainty is unavoidable without changing the real system, while epistemic uncertainty can be reduced by acquiring more knowledge about the system. When wanting to reduce the uncertainties in a model, one should focus on epistemic uncertainty only. Consequently, “epistemic uncertainty directly supports decisions about data collection and model improvement” [21].

3 Digital Twins and Uncertainty

DT’s features allow them to be a powerful tool for modelling and monitoring dynamic systems [34]. To begin with, a DT can collect data from the physical world in real-time, which could support the development of dynamic models [35]. Indeed, a DT could

provide at any moment information about the state of the physical system and assess the operational conditions of an uncertain situation. Moreover, the incorporation of data processing technologies such as artificial intelligence techniques into the DT may enable more effective predictive analysis and decision-making processes for uncertain events [36]. For this reason, in the next two sub-sections, we present an explicit overview of the aleatoric and epistemic uncertainties. First, we exploit the role of the DT in a dynamic system prone to uncertain events through literature. Then, we examine uncertainties within the DT model.

3.1 Uncertainty with Digital Twin in Dynamic Systems (Aleatoric)

There are numerous articles on asset management under uncertainties. A recent study in structural damage detection [37] describes DT implementation procedures with an approach combining physics-based models and machine learning. To quantify the uncertainties and develop a more robust method, a probabilistic approach in which a stochastic model is built and calibrated during the offline phase. In the same field, a technique for intelligent mission planning based on the DT is presented [38] while quantifying the uncertainties in the three components of the approach namely damage diagnosis, damage prognosis, and mission optimization considering aleatory and epistemic uncertainty sources using probabilistic methods. The structural health condition of an aircraft is known for its variability from one aircraft to another due to material qualities, mission history, pilot variability, and so on. The concept of dynamic Bayesian networks (DBN) is used in [39] to develop a health monitoring model for the diagnosis and prognosis of each aircraft. A probabilistic DT aircraft model is designed to achieve the integration of diverse uncertainty sources during the whole life of the aircraft wing, and decrease uncertainty in model parameters.

In the domain of safety and risk assessment, the authors of [34] emphasize how the inherent capability of DT in acquiring data from the physical world combined with the potentiality of reliable data processing could support a more effective diffusion of dynamic risk assessment models for improving safety. In this context, the authors of [40] propose the use of DTs to better identify high-risk scenarios and thus improve the risk assessment process. Another study [41] presents a framework for a DT based on machine learning and prognostics algorithms model to assess and predict the risk probability rate of an oil pipeline system by focusing on detecting a failure precursor and calculating risk conditions to determine remaining usable life (RUL).

In the manufacturing sector, several active areas can be identified that suffer from uncertainties, one of which is dynamic scheduling. Because of unknown occurrences in the manufacturing process, such as new task insertions, order cancellations, worker absences, and machine malfunctions, dynamic scheduling methods are needed and crucial to manufacturing systems. DT can aid in the detection of disruptions by continually comparing physical and virtual space and activating a rescheduling policy directly afterward a disturbance. In this context, [42] proposes reinforcement learning; (RL)-driven DT allows for effective collaborative scheduling between production and maintenance departments and assists manufacturers in improving real-time decision-making under uncertainties. Similarly, to address the aforementioned issues, the study in [43] presents a DT-enhanced dynamic scheduling strategy. Real data from physical machines and

simulated data from their virtual models are fused to assist machine availability prediction. Furthermore, to identify disruptions effectively, the DT's virtual entity, which is constantly updated alongside its physical counterpart, is used as a dynamic reference to quantify the deviation of the real production from the expected plan. Another study [44] provides an architecture for robust scheduling that is applied to a flow shop scheduling problem and is connected with the field through the DT. To deal with uncertain events, a module is dedicated to the real-time estimation of the equipment's failure probability, using data-driven statistical models on collected data.

In material resource planning (MRP), DT is also sought to deal with uncertain events. The study [45] describes how to use DT to improve MRP. During planning, numerous MRP factors are unknown; machine learning is used to predict these parameters. Nonetheless, no prediction is flawless, and the unpredictability of some parameters may have a significant influence on the final result. Thus, the optimization methods and probability distributions are used to consider these uncertainties.

The authors of [46] use hidden Markov models to approximate uncertainties in the design of manufacturing systems DT. The suggested DT is made up of two parts: a model component and a simulation component. Using discrete states and associated transition probabilities, the model component creates a Markov chain that embodies the dynamics underlying the phenomena. The simulation component uses a Monte Carlo simulation approach to reproduce the events. [47] presents a DT-based methodology for simulating all the effects in an artifact-based machine tool calibration process, starting with the machine and its expected error ranges and progressing to the artifact geometry and uncertainty, artifact positions in the workspace, probe uncertainty, compensation model, and so on.

To conclude, most studies rely on probabilistic approaches to model uncertainties due to the inherent variability of dynamic systems and the occurrence of external disturbances (aleatory uncertainty).

3.2 Uncertainty Within the Digital Twin Model (Epistemic)

A system's model is an expression that approximates how the real system behaves. Regardless of how accurate a model is or how much data is provided, it will always be an imperfect and incomplete representation of the real system. Indeed, according to [30], there is always some discrepancy between the modeled and the real behaviours when modelling a dynamic process to monitor its behaviour. This gap is called residual resulting in epistemic uncertainty. Disturbances and noise compromise the accuracy and reliability of the DT to monitor the physical system correctly.

The sources of uncertainty may include: linearizing nonlinearities to simplify the model, calibrating the model poorly and causing errors in parameters and model form, and considering some parameter's value constant even though it might change because of external effects such (e.g., environmental effects), or parameters having different values in different units (tolerance).

To overcome the aforementioned uncertainties, the authors of [48] address the design of the DT model for machinery process deterioration. They present an updated technique based on parameter sensitivity analysis. In [45], machine learning is used to fine-tune the DT's parameters to reduce the model discrepancy. [50] suggest a reinforcement learning

approach to adjust the model and data errors. In addition to reinforcement learning, [51] uses unsupervised learning to detect anomalies coming from measurement. Other studies use fuzzy logic to improve the performance of the DT by updating it based on measurements [52].

Overall, regardless of the emphasis put on the importance of uncertainty quantification in the DT, and the efforts put to fill the gap between the DT model and the physical system, the literature in this context remains poor and lacks more research and elaboration to tackle the different source of uncertainties and enrich DT functionalities.

4 Methodology

Model uncertainty, noise measurement, external disturbance, and stochastic outcomes of real systems all contribute to the use of uncertainty in modelling and inference. Thus, this section illustrates the approaches used to overcome the two types of uncertainty described previously, whether it is in the DT model or the real system. Figure 1 summarized the following information. First, because of the real process and its inherent disruptions, a physical system fed with identical inputs may display output variability (aleatory uncertainty). Second, uncertainty within the DT may be classified into two types: those depending on model parameters, known as model-parameter uncertainties, and those generated by modelling errors (epistemic uncertainty). Strategies for dealing with stochastic behaviours have been developed based on the type of uncertainty targeted.

- **Stochastic approaches:** Also called probabilistic approaches, all share a common mathematical foundation. These approaches aim to anticipate the occurrence of uncertainties by modelling them with probability densities. To model stochastic transitions in system dynamics with a DT one can use Hidden Markov Models [46] or Artificial Intelligence methods based on Probabilistic Graphical Models, e.g., Bayesian Networks (BNs) or Dynamic Bayesian Networks (DBNs) [39].
- **Non-probabilistic approaches:** Other non-probabilistic methods have been used to quantify and handle uncertainties. The goal of these approaches is to minimize the effects of uncertainty; they usually deal with epistemic uncertainties using fuzzy logic [52], rule-based techniques, and expert systems[53]. In the field of artificial intelligence, reinforcement learning is also being used [46].
- **Robust approaches:** Given the noise and uncertainty inherent in the system's monitoring, robust approaches aim at decoupling nonzero residuals caused by modelling errors/noise and disturbances of the physical system. Instead of establishing assumptions about stochastic distributions, it is feasible to use inference techniques to reduce the noise [54] or machine learning techniques to learn the normal pattern considering modelling errors and noise and still detect anomalies coming from the physical system [55]. The main objective of this approach is to obtain a DT which can listen to aleatory uncertainty from the physical system without being affected by epistemic uncertainties (model and parameters uncertainty).

Figure 1 summarizes the different uncertainty types and the approaches used to quantify them. In green, we represented a physical system monitored with a digital twin.

The observed input is the same for both systems. We measure the difference between the observed output and the expected output; the difference is called the residual. In red are represented the aleatoric uncertainties of the physical system due to its variability or external uncertain events occurrence connected with stochastic approaches to deal with them. In yellow are shown the epistemic uncertainties of the DT due to parameters, modelling errors or measurement noise connected to non-probabilistic approaches to deal with them. And then robust approaches are drawn in purple, connected to both epistemic and aleatoric uncertainties.

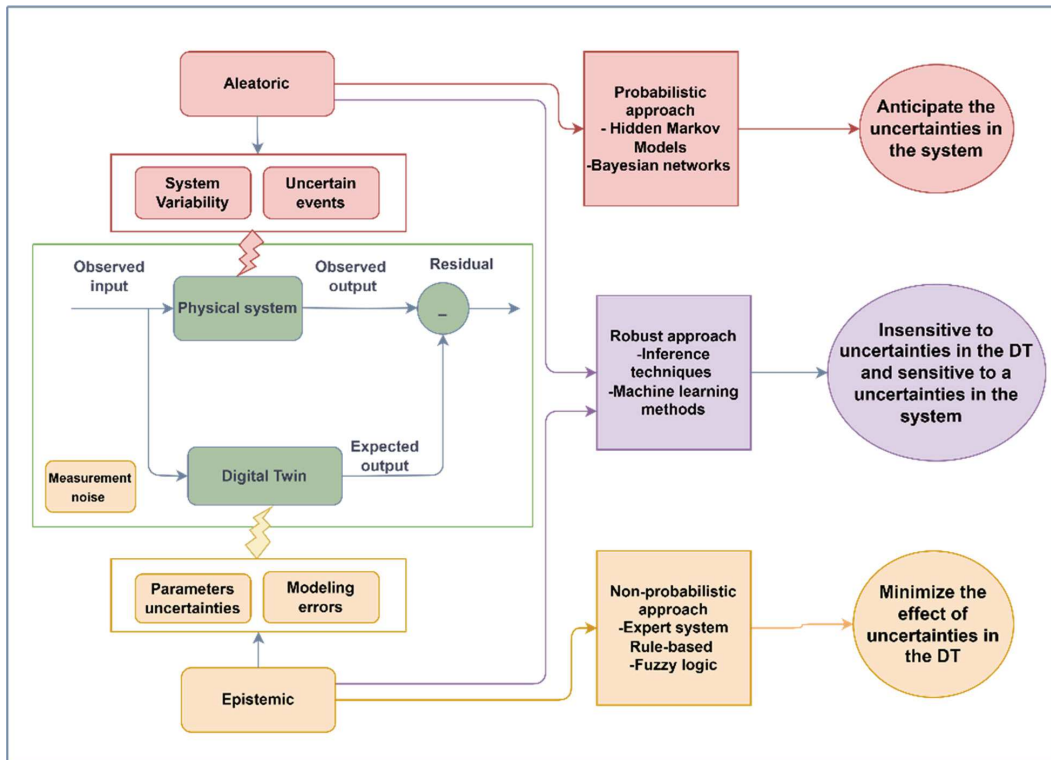


Fig. 1. Summary of uncertainty types and different used approaches to quantify them

5 Discussion

This article's contribution is twofold: (1) highlighting the capacity of a DT to quantify and manage uncertainties in dynamic systems, and (2) emphasizing the necessity of properly handling the uncertainty sources within the DT model itself. Overall, this article's outcome could help improve the use of DT in dynamic systems.

According to [56], uncertainty quantification is implicit in implementing a DT. That is, to allow risk-informed decisions, the best available knowledge about the DT features should be represented in the form of probability distributions or other uncertainty quantification metrics. However, as we saw in this article, aleatory uncertainty is irreducible whereas epistemic uncertainty can be reduced: the more knowledge we have about the real system, the less we need to model epistemic uncertainty. Indeed, if we have enough data from the physical system (e.g., by increasing the amount of data gathered and by

improving the data collection methods), there is no need to use probability distributions to model uncertain events. Thus, the DT would be fed in real-time by the occurring events and be synchronized with them. On the opposite, if we are not certain about the behaviour of the physical system, the model will approximate its behaviour through a probability distribution.

This is the main difference between the simulation and the emulation capabilities of a DT. A DT based on simulation approximates the behaviour of its environment with uncertainty quantification; it is not entirely in sync with the real world (static feature). On the contrary, a DT based on emulation only changes its state when it receives appropriate information from the real world; it is entirely in sync with the real world (dynamic feature) [12]. Thus, to reduce epistemic uncertainty, more efforts should be done to develop the emulation capabilities of a DT.

6 Conclusion

The ability of the Digital Twin to be updated in real-time has earned a lot of attention from both the scientific and industrial communities. As complex systems generally display stochastic behaviours, uncertainty should be a crucial part of DT modelling. However, many elaborations and works on the DT do not consider uncertainty. Thus, until today, the implementation of DT's ability to adapt to random events in the physical system is a core challenge. Two sources of uncertainties must be investigated: (1) the inherent variability of the studied system (aleatory), and (2) the lack of knowledge regarding the aforementioned system (epistemic).

In this sense, this paper first proposed a review of the concept of uncertainty. Then it clarified their presence in both the physical system and the Digital Twin model. Finally, it discussed the different methods used to estimate them. The paper's contribution is an overview of what has been written on the topic and the creation of the first basis for future research works.

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