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To cite this version:

HAL Id: hal-01743222
https://imt-mines-albi.hal.science/hal-01743222
Submitted on 7 Nov 2018

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MODELING OF CONTINUOUS SEWAGE SLUDGE DRYING IN A PADDLE DRYER BY COUPLING MARKOV CHAINS AND PENETRATION THEORY: INFLUENCE OF CONTACT AREA ESTIMATION

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Keywords: Sludge drying, Paddle dryer, Markov chains, Penetration theory

Abstract
Sewage sludge can be dried in paddle dryers and turned into an interesting material for energetic valorization. Matching the necessary water content at the end of the drying process for the different valorization pathways could lead to significant energy savings. However, the influence of operating conditions on process efficiency has not been studied extensively, and sludge rheology remains a scientific issue when it comes to mechanistic flow modeling. We thus developed and validated a model describing sludge flow in a continuous paddle dryer by a Markov chain. This flow model was coupled with the penetration theory, describing heat-transfer by conduction and water vaporization in an agitated packing. Both models are time-discretized and allow simulating water content and temperature profiles along the dryer at steady-state. The simulations are in good agreement with experimental water content profiles obtained on a lab-scale paddle dryer, with no adjustable parameters. Two approaches for estimating contact area between sludge and heated parts of the dryer are then compared. Our model better describes experimental data when the contact area is calculated in each cell rather than when it is considered constant and proportional to sludge hold-up in the dryer. However, this latter approach could be of practical importance for simulations of large-scale installation.
1- INTRODUCTION

Sewage sludge (SS) management is a matter of growing importance as its production keeps increasing with population growth and legislation restrictions. Storage and hygiene issues, as well as landfilling limitation, contribute to make drying a necessary step in sewage sludge treatment and valorization [1]. However, SS is a product quite difficult to handle, as it undergoes a transition to a pasty state during its drying [2]: this is potentially damageable for industrial dryers where moving equipment is involved. For this reason, paddle dryers are recognized as well adapted tool for SS drying [3]: thanks to the wedge-shaped paddles, the product undergoes high shearing stress and in situ clogging is avoided. Moreover, this kind of indirect dryer offers the advantages of compactness and low exhaust volumes. In spite of the important number of installations functioning at an industrial scale, design and operation of paddle dryers mostly rely on manufacturers’ know-how [4]. Few studies focused on the description of sludge drying in paddle dryers in continuous operation: while Arlabosse et al. [4] described drying kinetics along an industrial scale installation with a simple empirical model assuming plug flow of sludge, Tazaki et al. highlighted the importance of back-mixing in such installations via a Residence Time Distribution (RTD) study [5]. More recently, Charlou et al. developed a methodology for measuring experimental RTD with a good repeatability in a pilot-scale installation [6, 7]. This lead us to two major conclusions: water and dry solids have the same RTD, that is to say SS acts as a single phase during drying; moreover, this RTD can be described by a Markov chain, where each paddle represents a continuous stirred tank reactor (CSTR) [7]. These CSTRs exchange sludge flows in both directions, resulting in back-mixing: this is characterized by a recirculation coefficient \( R \) representing the ratio of recirculated sludge to the overall throughput in the reactor. This approach relies on the determination of transition probabilities between a cell and its neighbors during a given “transition time”, implying that time is discretized [8].

On the other hand, the so-called penetration theory, firstly developed in the 80s for the description of heat transfer and further for drying of granular free flowing solids, was successfully adapted to SS drying in batch lab-scale installations [9–11]. This theory postulates that, in such systems, drying and mixing can be described by a series of static periods during which transient heat transfer and water evaporation occur, separated by instantaneous perfect macro-mixing of the bulk [12, 13]. This theory gives access to the calculation of an effective heat transfer coefficient between the heated wall and the wet sludge, depending among others on sludge water content and bulk temperature. Sludge flow and heat transfer descriptions being based on time-discretization models, we can realize a coupling of these sub-models, which gives access to steady-state profiles of water content and sludge bulk temperature in a paddle dryer. This paper is dedicated to the description of the principle of this coupled approach and its comparison to experimental data, with a discussion on how the issue of estimating contact area with heated walls, a key parameter, should be dealt with.
2- MATERIALS AND METHODS

Description of the paddle dryer

The paddle dryer to be modeled is represented in Figure 1. It is composed of a U-shaped jacketed trough and a rotating shaft with up to 18 wedge-shaped paddles fixed on it. Trough and shaft are independently electrically heated and the paddles are heated up by conduction. SS is fed continuously by means of a Moineau pump, and superheated steam is used as a sweeping gas. Feed flow rate, discharge level, stirring speed, dryer slope and heating wall temperature can be varied.

![Figure 1: Scheme of the pilot-scale paddle dryer](image)

When a steady-state is reached during an experiment, it is possible to sample sludge along the dryer thanks to three traps located on top of the installation, which gives access to experimental water content profiles along the dryer. Moreover, at the end of experiments, sludge inlet and outlet are closed simultaneously in order to measure the dry solids hold-up when the installation has cooled down.

Sludge flow modeling by Markov chains

Sludge flow in the paddle dryer is modeled by two parallel Markov chains representing respectively dry solids and water in sludge. Each chain consists of \( n \) cells corresponding to the number of paddles. These cells are supposed to behave as a continuous stirred tank reactor (CSTR), with recycling of the flow between each cell due to the action of the paddles. This recycling is defined by the coefficient \( R \) representing the ratio of the recirculated flow to the overall sludge flow (Figure 2). The \( (n+1) \)th cell is an absorbing state, representing the outlet of the dryer, from which backwards transition is not possible.
Both chains are governed by the same matrix of \( n+1 \) by \( n+1 \) transition probabilities \( P \), since we have experimentally observed that water soluble and insoluble tracers have the same RTD. This matrix is tridiagonal, assuming that sludge may either stay in the same cell or go to one of the directly neighboring cells during each transition time \( \Delta t \): it is schematically presented in equation 1 for 3 cells, where \( P_{j,i} \) represents the transition probability from cell \( i \) to cell \( j \).

\[
P = \begin{bmatrix}
P_{i-1,i-1} & P_{i-1,i} & 0 \\
P_{i,i-1} & P_{i,i} & P_{i,i+1} \\
0 & P_{i+1,i} & P_{i+1,i+1}
\end{bmatrix}
\]  

At each transition, the evolution of a state vector \( S \) of length \( n+1 \) (for example mass of dry solid, mass of water, or enthalpy) is calculated according to equation 2, where \( S_{\text{feed}} \) is the vector corresponding to the quantity of \( S \) introduced in the system during \( \Delta t \).

\[
S(t+\Delta t) = P \ast [S(t) + S_{\text{feed}}]
\]  

The methodology for transition probabilities determination has already been described elsewhere [7]. The parameters governing the calculation of the \( P_{j,i} \) are \( R \), \( \Delta t \) and \( Hu \) the dry solids hold-up in the cell, i.e. the solids geometric residence time in the cell since the global throughput in the dryer is the same in each cell. In the following, we will consider that \( R \) is constant along the dryer, independent from sludge water content in each cell. From previous experiments we have seen that \( R \) values estimated by an optimization procedure generally lie between 2 and 5; in this range, this parameter has a minor influence on simulated RTD and no influence on the mean residence time. We thus set this parameter at \( R = 3 \). In a first approach, \( Hu \) is also considered constant along the dryer, i.e. the dry solids hold-up is equally divided in the cells: this hypothesis was made in our previous work where the experimental RTD could be described accurately by the Markov chains model. We also observed that experimental values measured for \( Hu \) (i.e. weighing of the whole dry solids hold-up at the end of an experiment divided by the number of cells) are very close to the optimized ones. However, this parameter has a great influence on the simulated RTD: we thus realized a design of experiments in order to establish a correlation between operating parameters and \( Hu \). In our range of operating conditions, it appeared that installation slope and discharge level were the only significant parameters influencing \( Hu \) [14]. In the following, we use this correlation for \( Hu \) estimation when comparing experimental data and simulated ones. The value of the last parameter \( \Delta t \) is discussed in the next section as it is common to both flow model and heat transfer model.

**Sludge drying and penetration theory**

The so-called penetration theory considers the continuous contact drying of an agitated wet granular packing as a succession of static periods and of instantaneous mixing of the bulk: during the static periods, transient propagation of a drying front occurs from the dryer wall to the free surface of the bulk [12]. This theory has proven efficient in describing the drying
kinetics of SS in a batch lab-scale installation, with the assumption that sludge can be considered as a saturated mono-dispersed packing of spheres [9]. We thus adapted this theory to our configuration: we keep the physical parameters for SS the same as in [9], and compute average heat transfer coefficients and evaporated water fluxes during $\Delta t$ in each cell of the Markov chain. The only varying parameters during a simulation are moisture content, temperature of the bulk and contact area between sludge and heated walls in each cell. The transition time of the Markov chain thus becomes equal to the fictitious static period in the penetration theory, which allows setting up a simple iterative algorithm leading to the description of steady-state temperature and water content profiles along the dryer (see below).

As highlighted in the first publications on the subject, the penetration theory relies on the determination of an empirical mixing number, $N_{mix}$. This number, which is critical for the determination of the static period $\Delta t$, can be estimated for paddle dryers according to equation 3 where $Fr$ is the Froude number [13]:

$$N_{mix} = 9 \times Fr^{0.05}$$

(3)

In our usual operating conditions, this leads to values of $\Delta t$ lying between 10 and 30 s according to equation 4, where $N$ is the stirring speed:

$$\Delta t = \frac{N_{mix}}{N}$$

(4)

As emphasized in [15], the correlation obtained by Mollekopf and presented in [13] may not be valid with any products or stirrer designs: the determination of an appropriate value for $N_{mix}$ is thus not straightforward. Another study [10] focused on sludge drying in a pilot-scale paddle dryer operated batch-wise: the authors obtained, after optimization, values for $N_{mix}$ relatively close to that given by equation 3, i.e. 6.5 vs. 8.5 respectively, for a stirring speed of 17rpm. Mollekopf’s correlation was also validated in a batch agitated indirect dryer for alumina spheres [9]. We then decided to keep this correlation for the calculation of characteristic mixing times in our model.

**Coupling of SS Markov flow model and penetration theory**

Both models rely on time discretization of continuous processes and are applicable to SS. Considering a common time-scale for these models allows a straightforward solution for computing steady-state sludge water content and temperature profiles along the dryer. Indeed, starting from a given state at time $t$ where dry solids hold-up, water content and bulk temperature of sludge are known in each cell of the dryer, the penetration theory allows calculating the amount of water evaporated and the new temperature of the bulk at $t+\Delta t$ in every cell. This model postulates that after $\Delta t$ an instantaneous macro-mixing occurs: at this step, we compute a new distribution for dry solids and water amounts as well as enthalpy in each cell according to Equation (2). Sludge is supposed to enter in the system at 100 °C. Enthalpies are calculated considering reference states for SS and water at $T_{ref} = 0$ °C as in Equation 5.

$$h(T) = m \times C_p \times (T - T_{ref})$$

(5)
Water content and bulk temperature are computed again before the next iteration; bulk temperature is deduced from Equation 6 (where \( w \) and \( ds \) indexes stand for water and dry sludge respectively).

\[
T = \frac{h_w + h_{ds}}{(m_w * C_{p,w} + m_{ds} * C_{p,ds})}
\]  

(6)

Starting from initial conditions, this iterative algorithm goes on until no more change is observed in temperature and water content profiles along the dryer. In this study, we considered numerical criteria as in Equation (7), where \( S \) stands for temperature or water content state vector along the dryer.

\[
\| S(t) - S(t - 20 \text{ min}) \| \leq 0.001
\]  

(7)

Once these criteria are matched, we consider that the system reached steady-state. During a transition, the amount of water evaporated in each cell is equivalent to the amount of water accumulated in that cell due to continuous sludge feeding, overflowing and recirculation between neighboring cells.

**Estimation of contact area between sludge and dryer heated walls**

As stated earlier, in our range of operating conditions, dry solids hold-up depends on operating conditions: this means that the dryer is not always filled to its maximum capacity. Consequently, the contact area depends on the amount of sludge in each cell. In a first approach, we considered that since the dry solids are homogeneously distributed along the dryer, the contact area would also be constant in each cell. We estimated that the dryer is full (shaft overlaid) when the quantity of dry sludge is \( m_{full} = 6 \text{ kg DS} \), corresponding to a contact area \( A_{full} = 1 \text{ m}^2 \). The contact area in the \( i^{th} \) cell \( A_i \) for a dry solids hold-up per cell \( Hu \) is then given by Equation (8).

\[
A_i = A_{full} * \frac{Hu}{m_{full}}
\]  

(8)

A second, more physical approach consists in calculating sludge volume in each cell at each time step, based on sludge and water amounts; the corresponding contact area is then deduced from this volume. The difficulty now lies in the estimation of SS density. As observed in the literature and previous works, SS is found in different states during its drying: in the first part of the dryer, it is in what is generally called a “pasty” or “lumpy” state, in which SS can be considered as a continuous material. At some point during the drying process, granulation of this continuum occurs as a result of the combined effects of drying and mechanical stirring [5, 10, 16, 17]. According to other studies conducted on paddle dryers, granulation occurs when sludge water content \( W \) is around 1.5 kg/kg, which is in agreement with our observations [10, 17]. As long as the water content is larger than this value, we calculate sludge density \( \rho \) according to Equation (9), where \( \rho_w \) is the density of water and \( \rho_{ds} \) is the density of dry sludge (in kg/m\(^3\)). We can make this assumption since it has been observed that sludge shows an ideal volumetric shrinkage in this range of water content, with a volume reduction equivalent to that of evaporated liquid water [18].
The estimation of sludge bulk density when it is in granular state is not straightforward, since it depends on particles mean diameter and on their water content. Due to lack of experimental data relating these parameters, we then consider that sludge bulk density decreases linearly with water content, from the value calculated previously for \( W = 1.5 \) kg/kg until the minimum value of 700 kg.m\(^3\) measured on a sample fully dried. Figure 3 illustrates the evolution of sludge bulk density obtained in this way on the whole range of water content considered in this study.

![Figure 3: Evolution of sludge bulk density with water content considered for the simulations with variable contact area along the dryer](image)

### 3- RESULTS AND DISCUSSION

#### Model validation

Model results were compared to experimental data gathered during a design of experiments: water content profiles were recorded once steady states were reached for different operating conditions [14]. Here, two examples are shown in order to illustrate the ability of the markovian model to represent SS drying in our continuous paddle dryer. Operating conditions and model parameters are gathered in Table 1. Geometric residence time \( \tau \) calculated according to Equation 10 (where \( Q_{ds} \) is the dry solids flow rate in g/h and \( n \) is the number of cells) for dry solids is also indicated with model parameters.

\[
\tau = \frac{H_u^* n}{Q_{ds}}
\]  

(10)
Table 1: Operating and modeling parameters for experiments and simulations performed

As can be seen in Table 1, experiments A and B were conducted with different stirring speeds and installation slopes, as well as slightly different sludge initial water contents. These differences influence model parameters as well, with $\Delta t$ depending on stirring speed and $Hu$ on the slope; consequently, geometric residence times are also very different. Based on these parameters, simulations were performed and the results are compared to experimental water content profiles in Figure 4. As expected, sludge final water content is much less for experiment B, for which $\tau$ is almost 3 times higher. The markovian model leads to similar trends in both cases, in spite of the relatively scattered experimental data. Moreover, the final water content is well predicted. A breakpoint appears at the beginning of the calculated profiles because we represented the initial water content at the position of 0 cm. Actually the first cell, which corresponds to the breakpoint, should have a higher water content: the hypothesis of sludge flowing in the system at 100 °C results in an overestimation of evaporation fluxes in this cell and thus the following to a lesser extent.

![Figure 4: Comparison between experimental water content profiles (o) and simulated ones (-) with the markovian approach: experiments A (left) and B (right)](image)

We do not have reliable experimental sludge temperature profiles. However, we compared on Figure 5 the results of our model to the ones obtained with the penetration theory applied to a batch reactor [9]. Both bulk temperatures as well as averaged heat transfer coefficient obtained in each cell during steady-state are in good agreement with the penetration model.
alone used with similar parameters. These results were obtained with the approach consisting in calculating the contact area at each time step for each cell. Such an approach allowed us validating our model, but is more time-consuming and not easy to implement to industrial-scale paddle dryers with more complex geometries.

Figure 5: Comparison of the temperatures (left) and heat transfer coefficient (right) as functions of sludge water content, obtained with the markovian model along the dryer (for each cell) (◊) and with the penetration theory for batch sludge drying in similar operating conditions (−); data obtained with parameters corresponding to experiment B

Comparison of the two approaches for contact area estimation

In this section, we compare the results, obtained with the two approaches described earlier for contact area estimation, with experimental data. The results obtained by considering a constant contact area in the cells, proportional to $Hu$ are labeled “Constant area”; the results obtained by computing the contact area at each time step are labeled ‘Variable area’. In all the simulations presented in Figure 6, the variable area approach leads to satisfactory agreement with experimental data. This is not the case with the constant area approach: for the lower values of $Hu$, i.e. the graphs on the left column, the model overestimates the water content profiles. On the contrary, for $Hu$ values higher than 100 g, the two approaches lead to similar results. This is due to Equation 8, in which we consider that the total contact area is proportional to the dry solids hold-up: this is probably true when the paddle dryer is close to being fully-loaded. However, it is not accurate when we work in operating conditions where hold-up and thus sludge level are low. In such cases, as in the left column graphs in Figure 6, the contact area is actually important because of the U-shape of our dryer.
Figure 6: Comparison of experimental water content profiles (o) and simulated ones obtained with a variable contact area approach (-) and a constant contact area approach (--); dry solids hold-up values per cell are indicated on the different graphs.

Another interesting aspect of this comparison is that the variable area approach considers that all cells have contact areas depending on their content, contrary to the constant area approach. If one considers the simulations on the right column of Figure 6, it appears that this distinction is not really influencing the model results. The total contact areas are really close, and the back-mixing softens the differences that could appear due to these different profiles of contact area per cell along the dryer. This observation is of practical importance, since industrial-scale paddle dryers are generally operated at their full capacity, with a maximum of
their potential contact area covered with sludge. In such a configuration, and with multi-shafts installations, the hypotheses of constant contact area, proportional to the dry solids hold-up would make more sense than trying to calculate an accurate value of contact area for each cell.

4- CONCLUSIONS AND OUTLOOKS

We propose a predictive model for sewage sludge drying in continuous paddle dryers. It relies on the coupling of a Markov-chain flow model with the penetration theory, a model for heat-transfer and water vaporization in agitated granular packing. Both models being time-discretized, we use the characteristic penetration time as a common time-scale for the two processes. An iterative algorithm leads to water content and temperature profiles during steady-state for given operating conditions. Two approaches were tested for contact area estimation between sludge and the heated parts of the dryer. The “Variable area” approach, which consists in calculating the volume of sludge and deducing the contact area for each cell at each time step, leads to good agreements between experimental and simulated data, allowing the validation of our model. However, a simpler and less time-consuming approach is obtained by considering a constant area per cell along the dryer: this approach is equivalent to the former when the lab-scale dryer is sufficiently loaded, and would be more easily applicable to industrial-scale paddle dryers. Future works will focus on the relations between sludge water content and its physical properties: the aim is to estimate R as function of water content and not considering it as an arbitrary parameter. More complex flow behaviors will also be considered, with the characterization of radial mixing in addition to the axial one considered alone as for now.

NOMENCLATURE

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<th>Symbols</th>
<th>Description</th>
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<tbody>
<tr>
<td>$A_i$</td>
<td>Contact area between sludge and heated walls in $i^{th}$ cell</td>
<td>m²</td>
</tr>
<tr>
<td>$A_{full}$</td>
<td>Maximum contact area in our paddle dryer</td>
<td>m²</td>
</tr>
<tr>
<td>$C_p$</td>
<td>Specific heat capacity</td>
<td>J.kg⁻¹.K⁻¹</td>
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<tr>
<td>$H_u$</td>
<td>Dry solids hold-up per cell</td>
<td>kg</td>
</tr>
<tr>
<td>$m$</td>
<td>Mass per cell</td>
<td>kg</td>
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<tr>
<td>$m_{full}$</td>
<td>Total dry solids hold-up when the dryer is full</td>
<td>kg</td>
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<tr>
<td>$n$</td>
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<td>$Q$</td>
<td>Sludge inlet flow rate</td>
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<td>$R$</td>
<td>Recirculation coefficient between neighboring cells</td>
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<td>$S$</td>
<td>State vector of length $n$</td>
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<td>$T$</td>
<td>Temperature</td>
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<tr>
<td>$W$</td>
<td>Gravimetric water content in water per dry solids mass</td>
<td>kg.kg⁻¹</td>
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<table>
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<tr>
<td>$\rho$</td>
<td>Specific density</td>
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<tr>
<td>$\tau$</td>
<td>Geometric residence time</td>
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<tr>
<td>$\Delta t$</td>
<td>Transition time</td>
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Subscripts

ds  Dry solids
w  Water

REFERENCES
