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MODELING OF SEWAGE SLUDGE FLOW IN A CONTINUOUS PADDLE DRYER

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Abstract

A model based on the theory of Markov chains has been developed to represent the residence time distribution (RTD) of municipal sewage sludge in a continuous paddle dryer. The flow of dry solids is described by a chain of n perfectly mixed cells, n corresponding to the number of paddles attached to the shaft. The transition probabilities between the cells are governed by two parameters: the parameter of internal recirculation, R , and the solids hold-up, Hu . In the absence of available correlation, both parameters are identified by fitting the model to experimental RTD data. The model demonstrates its ability to describe the sludge flow in a continuous lab-scale paddle dryer. A sensitivity analysis highlights that R is critical for the treatment uniformity while Hu controls the mean residence time and thus the final moisture content.

KEYWORDS: Stochastic approach, Markov Chains, Cells model, Residence time distribution, Experiment

INTRODUCTION

Sewage sludge management is an important issue for large urban areas. Considering their high organic content, municipal sewage sludge is a promising feedstock for Waste-to-

Energy (WtE) processes. Incineration is by far the most common process but pyrolysis and gasification emerge^[1–3]. Unfortunately, organic sludge has a high water content^[4]: it is usual to meet sludge with only 18 to 25% of solids after mechanical dewatering by centrifugation or belt filters. Higher solids contents (around 30 to 35%) can be achieved with filter presses but their discontinuous operation makes them unsuited for large wastewater treatment plants (WWTP). These solids concentrations are too weak for an efficient WtE conversion^[4]. Indeed, dry solids contents from 60% to 90% are expected for co-incineration with municipal solid wastes while more than 85% are needed for pyrolysis and gasification. As a consequence, drying is essential to increase the dry solids content prior to thermal valorization. Even if many technologies are available^[4–7], this operation remains an economical bottleneck. Paddle dryers offer several advantages such as high heat fluxes, low specific energy consumption and low exhaust volumes. Moreover, the design of the shafts is well adapted to overcome the sticky state and process the sludge up to complete drying without recycling dry matter upstream. However, in two thirds of the drying plants the dry solid content often exceeds 90 %^[8]. In addition to the lost energy, this excessive drying causes troubles for handling and storage of the dried sludge, as well as safety problems^[9]. Adjusting the solid content of the dried sludge to the valorization routes is essential to improve the sustainability of WtE conversion processes.

Until now, few efforts have been dedicated to experimental and numerical investigations on continuous paddle dryers for sludge treatment. Whereas tools to describe, and even predict, heat and mass transfers have been available for a few years^[10–12], sludge flow

has been partly characterized very recently. According to Tazaki et al.^[13] and Charlou et al.^[14], the flow pattern is not a simple plug flow. Back mixing of the sludge occurs^[13], especially near the middle of the dryer where the sludge consistency is sticky. To our knowledge, the modeling of sludge flow has not been addressed in the literature yet. Sludge rheological behavior is complex^[15–17] and poorly documented for high dry solids contents^[15]. As a result, mechanistic approaches and computational fluid dynamics (CFD) modeling are out of reach for the time being. Chemical engineers use the concept of residence time distribution (RTD) to characterize flow patterns in a reactor^[18, 19]. The principle of RTD measurements is to inject an inert tracer into the reactor at some time $t=0$ and to measure the tracer concentration in the outlet stream as a function of time. The residence time distribution function, which describes in a quantitative manner how much time different fluid elements have spent in the reactor, is computed from the concentration-time curve. Based on these experimental data, a flow model can be proposed to predict the RTD in the reactor or the apparatus. Coupled with kinetic data, the average outlet concentration in a real reactor or apparatus can then be predicted. Many models are available to describe deviations from the ideal continuous stirred tank reactor (CSTR) or plug flow reactor (PFR)^[20]. Cell models, mostly based on the theory of Markov chains^[21], present the advantage of describing the design of the apparatus, since the cell size can be conditioned by sections covering the paddles for instance. Markov chains proved to be efficient to model bulk particulate flow in a great variety of processes^[22–25]. This approach has been widened to describe heat and mass transfers between stochastically moving particulate and gas flows^[26, 27]. In this paper, we assume that the agitated contact drying process is a Markov process and we propose a cell model,

based on a 1D homogeneous Markov chain, to describe the flow of the anhydrous solids in a paddle dryer. Experimental RTD data are used to identify the two models parameters. A sensibility study of the model responses – the mean residence time and the variance of the RTD - to the variations of the model parameters is also undertaken to identify the most influential model parameters.

MODEL DESCRIPTION

Physical Model And Modelling Assumptions

In this study, a lab-scale continuous agitated contact dryer, designed and built in our institute (see Figure 1), is used as an example to build the physical model. A full description of the experimental set-up is available in Charlou et al. ^[14]. The main characteristics are summarized hereafter. The dryer consists of a U-shaped heated trough with one rotating shaft inside. Nineteen full wedge-shaped paddles are screwed onto the rotor and heated by conduction. In the current configuration, the paddles are regularly distributed along the shaft. Scrapers, formed to interact with the paddles and generate intense shearing, are positioned on the trough. The sludge is fed into one end of the trough and flows due to gravity, dryer slope and mechanical agitation to the opposite end, where the dry material is discharged. The sludge structure evolves from a fluid to a pasty and, then, to a particulate matter.

The principle of the Markov model is to represent the solids circulating in the dryer by a chain containing a finite number of cells. Let us divide the total length of the process into n perfectly mixed cells of equal length and mass of dry matter. As we want to preserve a

physical description of the process, n is not an adjustable parameter of the model. In the following, we will assume that the number of active cells, n , is equal to the number of paddles. When a granular flow exists, we can assume that, in the vicinity of the paddle, part of granules will flow forward whereas the other part will stay in the vicinity of the paddle or will move backward. Thus, in the presence of recirculation phenomena, the cells must be connected by internal flows, which represent the circulation of the matter forward (from the current cell to the following one) and the recirculation in the opposite direction (from the current cell to the previous one). We will consider that only transitions between neighboring cells are possible between two successive observations of the system. At the end of the chain, a last cell ($n+1$) must represent the collector of dried sludge particles located at the dryer outlet. This cell is called the absorbing cell. Of course, backward transition from the absorbing state is forbidden. The structure of the 1D chain is presented in Figure 2. We will investigate exclusively the steady state operation of the dryer.

Mathematical Model

Let us observe the state of the process at discrete moments of time $t_m = m \cdot \Delta t$, where Δt is the transition duration between two successive observations, and m is the number of transitions. If we examine the evolution of any additive property S (mass, for instance) of a flow, its distribution over the process length can be presented as a state vector

$\mathbf{S} = [S_1 \ S_2 \ S_3 \ \dots \ S_n]^t$. During Δt , the state S_i in the i -th cell can change due to the amount of S that comes from the other cells or that leaves the cell. Thus, the evolution of S is controlled by a matrix of transition probabilities $\mathbf{P}$. Δt has to be small enough so that

only transitions between neighboring cells are possible. As a result, three probabilities can be considered: a probability of staying in the cell, $P_{i,i}$, a probability to move forward, $P_{i+1,i}$, and a probability to move backward, $P_{i-1,i}$. The matrix $\mathbf{P}(n+1 \times n+1)$, that contains all the possible transition probabilities corresponding to a transition time Δt , takes the form described by Eq.(1).

$$\mathbf{P} = \begin{bmatrix} P_{1,1} & P_{1,2} & \dots & 0 & 0 & 0 & 0 \\ P_{2,1} & P_{2,2} & \dots & 0 & 0 & 0 & 0 \\ 0 & P_{3,2} & \dots & P_{i-1,i} & 0 & 0 & 0 \\ \dots & \dots & \dots & P_{i,i} & \dots & \dots & \dots \\ 0 & 0 & 0 & P_{i+1,i} & \dots & P_{n-1,n} & 0 \\ 0 & 0 & 0 & 0 & \dots & P_{n,n} & 0 \\ 0 & 0 & 0 & 0 & 0 & P_{n+1,n} & P_{n+1,n+1} \end{bmatrix} \quad (1)$$

In steady state operation, the condition of mass conservation involves that the sum of all the elements in each column is equal to one. As the backward transition from the absorbing state is forbidden, $P_{n+1,n+1} = 1$.

According to the stochastic motion of the particles, the throughput in a cell is not equal to the throughput in the dryer. The difference is taken into account by the recirculation coefficient R which is defined as the ratio between the recirculated flow in each cell and the inflow rate (see Figure 2).

The geometrical residence time, τ_{cell} , and the probabilities $P_{i,i}$, $P_{i+1,i}$, $P_{i-1,i}$ are defined in Table 1. Hu is the dry solids hold-up in each cell and Q_{ds} is the dry solids throughput in the Markov chain. During the transition time Δt , the probability of staying in a cell is an

exponential function of this time ^[24]. The matrix of transition probabilities $\mathbf{P}$ is constant since the model describes a steady-state operation. In Markovian language, the process is said linear and the Markov chain homogeneous. Moreover, we assume that R and Hu do not vary along the cells in a first approach, since we cannot estimate experimentally the variation of these parameters. The evolution of the state vector for a transition Δt is then described by the following formulas:

$$\mathbf{S}(t + \Delta t) = \mathbf{P} * \mathbf{S}(t) \quad (2)$$

or

$$\mathbf{S}(t + \Delta t) = \mathbf{P}^m * \mathbf{S}(t) = 0 \quad (3)$$

Numerical Model

Using the RTD concept, the experimenter gets the concentration-time curve by injecting a tracer in a known concentration (or quantity) at the reactor inlet and analyzing this tracer concentration (or quantity) in the product at the outlet. Then he computes the RTD function, $E(t)$, which describes in a quantitative manner how much time different fluid elements have spent in the reactor. For a Dirac injection, i.e. instantaneous and not perturbing the flow, $E(t)$, also called the exit age distribution function, is given by Eq. (4), while its cumulative form $F(t)$ is given by Eq. (5):

$$E(t) = \frac{C(t)}{\int_0^\infty C(t) dt} \quad (4)$$

$$F(t) = \int_0^\infty E(t) dt \quad (5)$$

Numerically, we can simulate the Dirac injection of a tracer in the system by using the following initial state vector, which means that at $t=0$, a tracer of quantity 1 is injected in the first cell of the system:

$$\mathbf{S}_{t=0} = \begin{bmatrix} 1 & 0 & 0 & \dots & 0 \end{bmatrix}^t \quad (6)$$

$E(t)$ is calculated as the difference, between two successive transitions, of the tracer quantity reaching the absorbing cell as in Eq. (7), which corresponds to a discretized version of Eq. (4). $F(t)$ is given by the amount of tracer cumulated in the absorbing state (Eq. (8)):

$$E(t) = S(t)_{n+1} - S(t - \Delta t)_{n+1} \quad (7)$$

$$F(t) = S(t)_{n+1} \quad (8)$$

Knowing $E(t)$, it is easy then to compute the mean residence time, t_s , the variance, σ^2 , and the dimensionless variance, σ_D^2 :

$$t_s = \int_0^{\infty} t \cdot E(t) dt \quad (9)$$

$$\sigma^2 = \int_0^{\infty} (t - t_s)^2 E(t) dt \quad (10)$$

$$\sigma_D^2 = \left(\frac{\sigma}{t_s} \right)^2 \quad (11)$$

The model only depends on two parameters: the recirculation coefficient, R , and the ratio $\xi = Hu/Q_{ds}$ between the dry solids hold-up and the dry solids throughput. Both parameters are a priori unknown. But, if we consider that usually the dry solids throughput Q_{ds} is known, only the solids hold-up, Hu has to be identified. A third parameter, the transition duration, Δt , also appears in the mathematical model. Previously, we assumed that the transition duration, Δt , is small enough so that particles in a cell could be able to transit only to the neighboring cells. But the choice of Δt is not straightforward. Moreover, only some scarce papers explain how the choice was made [22, 28]. Consequently, the influence of the transition duration on the mean residence time and the dimensionless variance was investigated numerically. The selection of Δt has to be done by taking into account that the smaller Δt the higher the model precision, but the higher the calculation time. Figure 3 illustrates the evolution of the standardized mean residence time and the standardized dimensionless variance according to the transition time for $R=1.9$ and $\xi=0.19$ h or $R=4.6$ and $\xi=0.16$ h. The computed values were standardized by the values for the lowest transition duration, i.e. $\Delta t=0.5$ s. For $\Delta t>3.6$ s, the mean residence time and the standardized dimensionless variance strongly depend on the value of the transition duration. Below this threshold, they tend gradually towards a constant value and the exit age distribution function, $E(t)$, does not change anymore (see Figure 4). For $\Delta t=2$ s, both RTD characteristics are in the 99% confidence interval. To get a compromise between model precision and calculation time, we selected $\Delta t=2$ s.

In the following, the recirculation coefficient R and the solids hold-up Hu are identified by fitting the model to the experimental RTD data using MATLAB[®] software. The

fminsearch function (based on a Nelder-Mead algorithm) was used to minimize the least sum of squares of the difference between experimental and simulated $E(t)$. A threshold of 10^{-4} was set for the variation of the residue, ΔE , between two successive iterations. In Eq. (9) the subscripts *exp* and *cal* stand for the experimental and corresponding calculated discrete values of $E(t)$ respectively.

$$\Delta E = \sum (E_{exp} - E_{cal})^2 \quad (12)$$

RESULTS AND DISCUSSION

Ability Of The Cells Model To Describe Sludge Flow In A Lab-Scale Paddle Dryer

Experiments were performed on the lab scale paddle dryer using the sewage sludge from the municipal WWTP of Albi (France) as feedstock. The WWTP implements a conventional extended aeration process, a nitrification/denitrification process, a biological phosphorus removal and a mesophilic anaerobic digestion. The sludge was sampled after centrifugation and stored at 4 °C in tight containers during a minimum of 48 h before the experiments. Its dry basis initial moisture content was 3.7 kg/kg.

Drying experiments were performed at atmospheric pressure, with a dryer wall temperature set at 160 °C and with 1.1 kg/h of superheated steam at 160 °C as sweeping gas. The dryer was tilted of 3° and the inlet mass flow rate was set at 2.5 kg/h on a wet basis. When the steady state was reached, 3 successive pulse injections of tracers were performed at 1 h interval each. Partially dried sludge was sampled at the outlet at a frequency of 5 min, dried at 105 °C for 24 h and analyzed off-line using X-ray

fluorescence spectroscopy (PANalytical, Almelo, Netherlands), which give access to experimental $E(t)$ curves^[29].

As mentioned before, the dry solids throughput Q_{ds} is known indirectly, from the inlet mass flow rate and moisture content. Thus, only the recirculation coefficient, R , and the solids hold-up, Hu , have to be identified in our model. Figure 5 illustrates the ability of the model to represent the experimental RTD function, $E(t)$, and its cumulative form $F(t)$ after the optimization described previously, for one of the three injections realized. For the experiments, the stirring speed was set to 28 rpm (see Figure 5 a) and 14 rpm (Figure 5b) respectively. For both experiments, the model describes experimental data with a good accuracy.

The parameters from the experimental data and their mean values calculated after optimization of the three modeled RTD curves are reported in Table 2 for both experiments. Experimentally, the solids hold-up is obtained by weighing the total remaining dry sludge in the dryer at the end of an experiment. The experimental and optimized values of Hu are close in both cases, indicating that the optimization procedure does not end in unrealistic values of this physical parameter. Finally, we observe a slight difference between R values obtained for both experiments, but this difference is not really significant with respect to the standard errors. Since both RTD curves have similar shapes in spite of these slight parameters differences, the respective influences of R and Hu on model results should be studied before concluding on the effect of stirring speed on these parameters. For this reason, we proceeded to a sensitivity analysis of the model.

Sensitivity Analysis

To clarify the influence of both parameters on the RTD, a numerical parametric study was performed in the range $0 \leq R \leq 20$ and $0.025 \leq \zeta \leq 0.18$ h. Figures 6 and 7 emphasize the influence of R and ζ on the RTD curves while Figure 8 highlights the influence of both parameters on the mean residence time and the dimensionless variance. R has little influence on the mean residence time. An increase of R results in a strong enhancement of the dimensionless variance. As can be seen on Figure 6, the exit age distribution function spreads out when R increases and the flow evolves from the ideal plug-flow ($R=0$) to the one observed in an ideal continuous stirred tank reactor. From a practical point of view, it is essential to reduce back-mixing phenomena in paddle dryers ($R \rightarrow 0$) to achieve a uniform drying, through the shaft retrofit for instance. On the contrary, ζ has a strong influence on the mean residence time. The evolution is almost linear: the mean residence time increases by a factor 4.4 when ζ is multiplied by 4.5. Thus, the solids hold-up is the most critical parameter to control the moisture content at the dryer exit. For the time being, no correlation is available in the literature to predict the solids hold-up from the operating conditions.

CONCLUSIONS

A cell model, based on the theory of Markov chain, has been developed to describe sludge flow in a paddle dryer. To preserve a physical description of the process, the number of cells corresponds to the number of paddles fixed on the dryer shaft. The model is able to predict the residence time distribution, the mean residence time and the variance. Only transitions between neighboring cells are allowed between two successive

observations of the system. The model only depends on two parameters: the recirculation coefficient and the volume of the cells, i.e. the solids hold-up in the dryer. The recirculation coefficient has an influence on the distribution width, and thus on the uniformity of the drying treatment. The solids hold-up in the dryer is critical for the mean residence time of solids in the dryer and the control of the outlet moisture content. We demonstrate the ability of the model to describe experimental RTD curves achieved on a lab-scale continuous paddle dryer. But, in the absence of correlation in the literature to predict the solids hold-up in the dryer, both parameters have to be identified by adjusting the model to the experimental data. The establishment of this correlation, for at least the lab scale process, is in progress. The extension of this model to predict moisture profile along the dryer, by coupling a heat and mass transfer model based on the penetration theory, is also investigated.

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NOMENCLATURE

C	Tracer concentration (kg.kg^{-1})
$E(t)$	Exit age distribution function (h^{-1})
$F(t)$	Cumulative function (-)
m	Number of transition (-)
Q_{ds}	Dry solids mass flow rate (kg.h^{-1})

Hu	Dry solids hold-up (kg)
n	number of active cells (-)
$\mathbf{P}$	Matrix of transition probabilities (-)
$P_{j,i}$	Probability to move from cell i to cell j (-)
R	Recirculation coefficient (-)
$\mathbf{S}$	State vector (-)
t	Time (h)
t_s	Mean residence time (h)

Greek letters

σ^2	Variance (-)
σ_D^2	Dimensionless variance (-)
τ_{cell}	Geometric residence time in a cell (h)
ζ	Solids holdup to mass flow rate ratio (h)
ΔE	Minimization function (-)
Δt	Transition time (s)

Subscripts

$cell$	Cell of the Markov chain
D	Dimensionless
i	Cell number
exp	Experimental value
cal	Calculated value

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Table 1. Geometric residence time and transition probabilities in each cell

Cell	i=1	1<i<n	i=n	i=n+1
τ_{cell}	$\frac{Hu}{1+R Q_{ds}}$	$\frac{Hu}{1+2R Q_{ds}}$	$\frac{Hu}{1+R Q_{ds}}$	0
$P_{i,i}$	$e^{-\frac{\Delta t}{\tau_1}}$	$e^{-\frac{\Delta t}{\tau_i}}$	$e^{-\frac{\Delta t}{\tau_n}}$	1
$P_{i-1,i}$	0	$\frac{R}{1+2R} 1-P_{i,i}$	$\frac{1}{1+R} 1-P_{n,n}$	0
$P_{i+1,i}$	$1-P_{11}$	$\frac{1+R}{1+2R} 1-P_{i,i}$	$\frac{R}{1+R} 1-P_{n,n}$	0

Table 2. Experimental and calculated parameters from RTD curves in Figure 5

Parameter	t_s (h)	σ^2 (h ²)	Hu (g)	R (-)	Stirrer speed
Experimental	1.3	0.9	33.2	(-)	28 rpm
Optimized	1.2 ± 0.1	0.6 ± 0.1	31.4 ± 1.7	4.5 ± 1.2	
Experimental	1.3	0.6	31.6	(-)	14 rpm
Optimized	1.3 ± 0.1	0.5 ± 0.1	33.2 ± 1.4	3.3 ± 0.7	

Figure 1. Schematic representation of the lab-scale paddle dryer and photo of the mixing device.

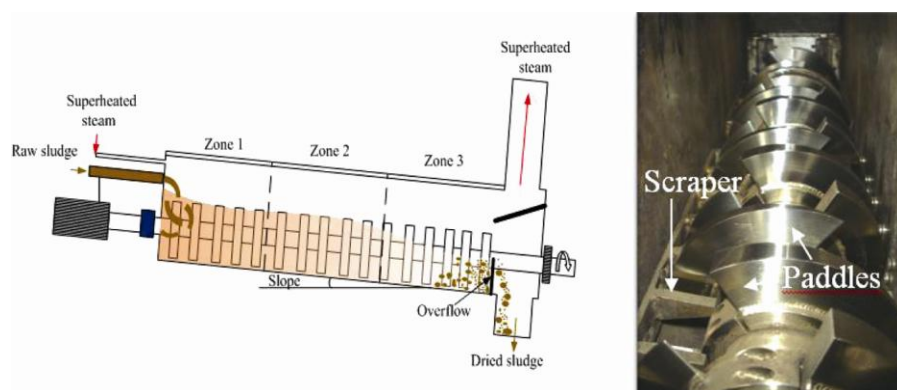


Figure 2. 1D cells chain for the modelling of solids flow in a paddle dryer.

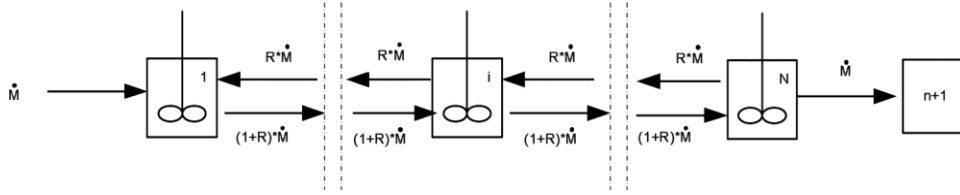


Figure 3. Influence of the transition duration, Δt , on the mean residence time and the dimensionless variance, standardized by the results for $\Delta t=0.5$ s, for $R=1.9$ and $\zeta=0.19$ h (dot) or $R=4.6$ and $\zeta=0.16$ h (square).

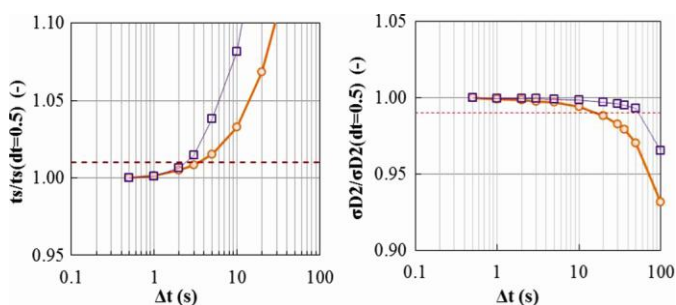


Figure 4. Influence of the transition duration, Δt , on the exit age distribution function, $E(t)$, for $R=4$ and $\xi=0.1$ h.

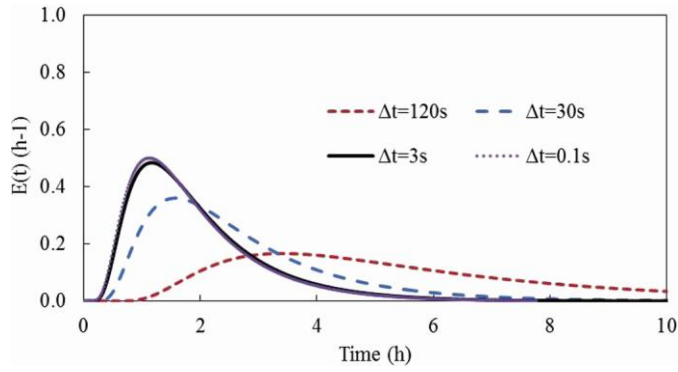


Figure 5 – Comparison of experimental and computed RTD functions for a pulse injection of tracer in steady-state operation (only one tracer is represented): (a) stirring speed of 28 rpm; (b) stirring speed of 14 rpm.

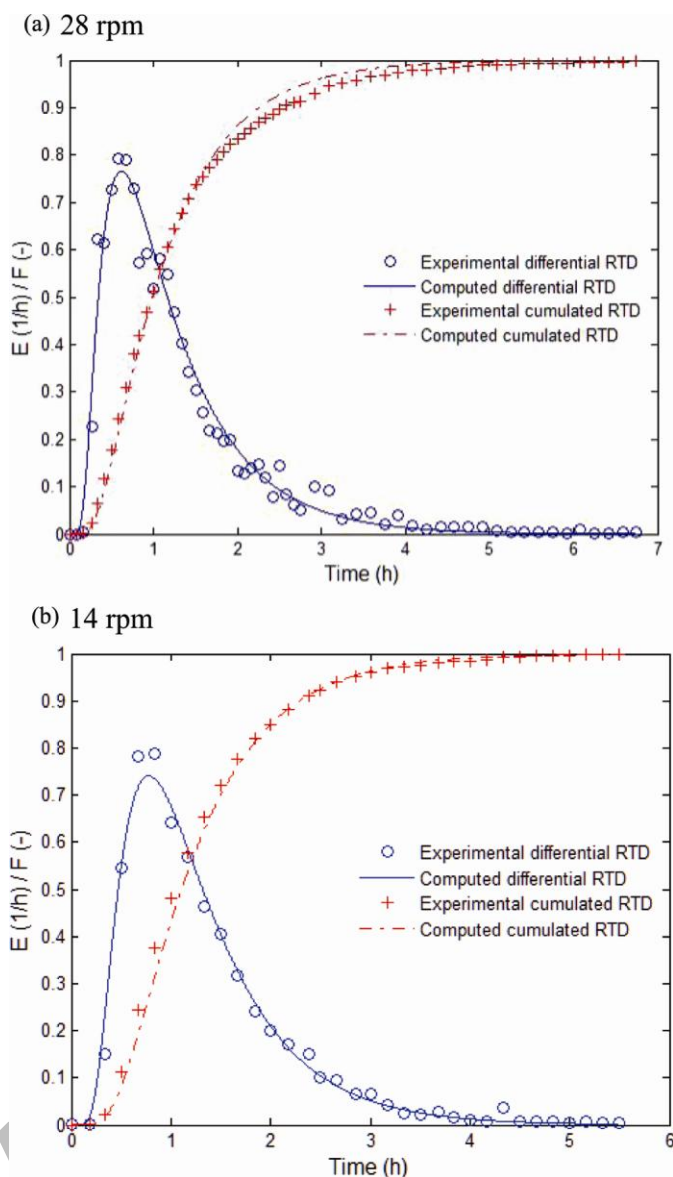


Figure 6. Influence of the recirculation coefficient R on the exit age distribution function

$E(t)$ for $\xi = 0.1$ h.

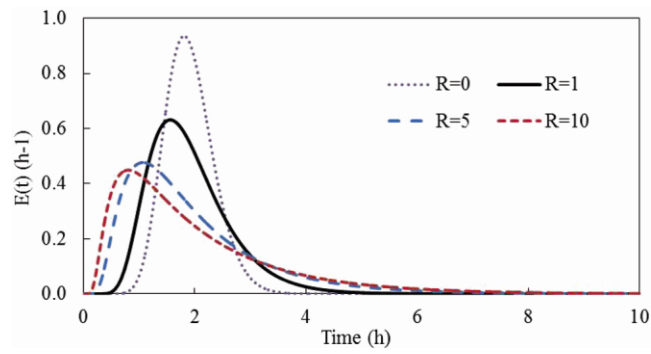


Figure 7. Influence of the ratio ξ on the exit age distribution function $E(t)$ for $R=4$.

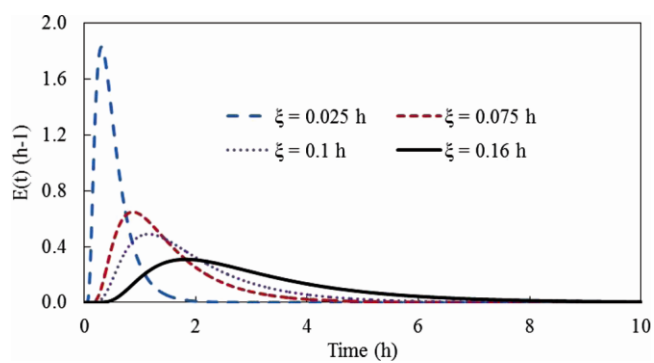


Figure 8. Influence of the recirculation coefficient R on the mean residence time (left) and the dimensionless variance (right) for different values of the ratio ξ .

